Chapter 7 Work and Kinetic Energy

Which one costs energy?

Question: (try it)
How to throw a baseball to give it large speed?
Answer:
Apply large force across a large distance!

Force exerted through a distance performs mechanical work.
Units of Chapter 7

• Work Done by a Constant Force
• Kinetic Energy
• The Work-Energy Theorem
• Work Done by a Variable Force (optional)
• Power

Read Chapter 8, Potential energy before the next lecture.
We will finish Chapter 8 in the next lecture.
7-1 Work Done by a Constant Force

When the force is parallel to the displacement:
Constant force in direction of motion does work $W$.

$$W = Fd$$  \hspace{1cm} (7-1)

SI unit: newton-meter (N·m) = Joule, J

1 J = 1 N.m

If $F = 15$ N, distance = 2 m, $W = 30$ J
7-1 Work Done by a Constant Force

1 Joule
1 J

How much is that?

<table>
<thead>
<tr>
<th>Activity</th>
<th>Equivalent work (J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual U. S. energy use</td>
<td>$8 \times 10^{19}$</td>
</tr>
<tr>
<td>Mt. St. Helens eruption</td>
<td>$10^{18}$</td>
</tr>
<tr>
<td>Burning one gallon of gas</td>
<td>$10^8$</td>
</tr>
<tr>
<td>Human food intake/day</td>
<td>$10^7$</td>
</tr>
<tr>
<td>Melting an ice cube</td>
<td>$10^4$</td>
</tr>
<tr>
<td>Lighting a 100-W bulb for 1 minute</td>
<td>6000</td>
</tr>
<tr>
<td>Heartbeat</td>
<td>0.5</td>
</tr>
<tr>
<td>Turning page of a book</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>Hop of a flea</td>
<td>$10^{-7}$</td>
</tr>
<tr>
<td>Breaking a bond in DNA</td>
<td>$10^{-20}$</td>
</tr>
</tbody>
</table>
If the force is at an angle to the motion, it does the following work:

\[ W = (F \cos \theta)d = Fd \cos \theta \]  

(7-3)

\( \theta \) is the angle between force and motion direction.

Pulling at \( \theta = 20^\circ \), \( F=15 \text{N} \), \( d=2 \text{ m} \)

\[ W = Fd \cos \theta = 15 \times 2 \times \cos 20 = 28 \text{ J} \]
7-1 Work Done by a Constant Force

The work can also be written as the dot product of the force and the displacement:

\[ W = \vec{F} \cdot \vec{d} = Fd \cos \theta \]

\( \theta \) is the angle between force and motion direction.
The work done may be positive, zero, or negative, depending on the angle between the force and the motion:

\[ W = (F \cos \theta)d = Fd \cos \theta \]

Here for F and d we only use their sizes (absolute value). The sign of the work is determined only by the angle between that force and motion.
\[ W = Fd \cos \theta \]

**Special cases:**

When force is perpendicular to motion direction, it does no work.

Examples:
- Normal force is always perpendicular to surface.
- Tension of pendulum …

When force is opposite to motion direction, \( \cos(180) = -1 \).

Examples:
- Kinetic friction force does negative work.
  \[ W_{fk} = -f_k \, d \]
  Always!
If there is more than one force acting on an object, we can find the work done by each force and add them together to find the total work.

Total work: \( W_{\text{total}} = W_1 + W_2 + W_3 + \ldots \) the sum of the work done by each force.
Q: Is Work a scalar or a vector? 
Scalar!
It only says how much energy is added or used, (positive or negative), but doesn’t indicate motion directions or force directions. 
When we add work to get total work, we add as positive and negative simple numbers. 
We don’t add work as vector arrows.

\[
\begin{align*}
\vec{f}_k & \quad \rightarrow \quad \vec{F}_{\text{pull}}
\end{align*}
\]

If \( F_{\text{Pull}} = f_k \); \( a=0 \), \( v=\text{constant} \)
\[ W_{\text{Pull}} = F_{\text{Pull}} \cdot d ; \quad W_{f_k} = -f_k \cdot d ; \quad W_{\text{total}} = 0 \]

If \( F_{\text{Pull}} > f_k \); \( a >0 \), If \( v >0 \), \( v \) will increase.
\[ W_{\text{total}} = F_{\text{Pull}} \cdot d - f_k \cdot d = (F_{\text{Pull}} - f_k) \cdot d = F_{\text{net}} \cdot d \]
7-2 Kinetic Energy and the Work-Energy Theorem

When positive work is done on an object, its speed increases; when negative work is done, its speed decreases.
7-2 Kinetic Energy and the Work-Energy Theorem

After algebraic manipulations of the equations of motion, we find: The total work done to one object is always equal to the change of its $\frac{1}{2}mv^2$

$$W_{\text{total}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Therefore, we define the kinetic energy:

$$K = \frac{1}{2}mv^2$$ (7-6)

Kinetic energy has SI unit: J
(same dimension as Work)
1 kg m$^2$/s$^2 = 1$ (kg m/s$^2$)m =1Nm =1 Joule
7-2 Kinetic Energy and the Work-Energy Theorem

Work-Energy Theorem: The total work done on an object is equal to its change in kinetic energy.

\[ W_{\text{total}} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 \]  

\[ W_{\text{total}} = K_f - K_i \]  

\[ K_f = K_i + W_{\text{total}} \]

It’s true for ALL MOTIONS, no only for constant a motion!!!
The problem solving strategy for work problems is as follows:

1. Compute work for individual forces first.
2. Add all work together as scalar numbers.
3. Set equation:
   \[ W_{\text{total}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \]

If you know total work, you can solve for \( v \); if you know \( v \), you can solve for work.

Given:
- \( m = 1000 \text{ kg} \)
- \( v_0 = 0 \)
- \( \mu_k = 0.2 \)
- \( \phi = 30^\circ \)
- \( d = 0.5 \text{ m} \)

Find \( v_f \)

\[
\begin{align*}
W_{mg} &= mg \cdot d \cdot \cos 60 = 9800 \cdot 0.5 \cdot \cos 60 = 2450 \text{ J} \\
W_{fk} &= -f_k \cdot d = -1697 \cdot 0.5 = -849 \text{ J} \\
W_N &= 0 \\
W_{total} &= 2450 - 849 = 1601 \text{ J} \\
K_f &= K_i + W_{total} \\
\frac{1}{2}mv_f^2 &= W_{total} = 1601 \text{ J} \\
v_f &= 1.79 \text{ m/s} 
\end{align*}
\]
Work is a scalar.

To get total work done by all forces, add work in Joules directly as simple numbers!

You only need to worry about the angles between each force and actual motion when you calculate work done by each force.

After the work is calculated. Add them as simple numbers, no worry about direction any more.

\[ W_{mg} = mg \cdot d \cdot \cos 60^\circ = 2450 \text{ J} \]
\[ W_{fk} = -f_k \cdot d = -849 \text{ J} \]
\[ W_{total} = 2450 + (-849) = 1601 \text{ J} \]
Spring Force: Hooke’s Law \[ F_{\text{spring}} = -k \Delta x \]

- \( k \): Hooke’s constant (how strong a spring is)
- \( \Delta x \): distance of stretch/compression

**Force direction:**
Always in the opposite direction of \( \Delta x \)

**Spring force** always tries to recover its natural Length

**Attention:**
This \( k \) is not \( K \), Spring constant is not Kinetic Energy.
7-3 Work Done by a Variable Force

If the force is constant, we can interpret the work done graphically:

\[ \text{Area} = Fd = W \]
7-3 Work Done by a Variable Force

If the force takes on several successive constant values:
7-3 Work Done by a Variable Force

We can then approximate a continuously varying force by a succession of constant values.
7-3 Work Done by a Variable Force

The force needed to stretch a spring an amount \( x \) is \( F = kx \).

Therefore, the work done in stretching the spring is

\[
W = \frac{1}{2} kx^2 \quad (7-8)
\]
7-4 Power

Power is a measure of the rate at which work is done:

\[ P = \frac{W}{t} \]  

(7-10)

SI unit: J/s = watt, W

1 horsepower = 1 hp = 746 W
# 7-4 Power

<table>
<thead>
<tr>
<th>Source</th>
<th>Approximate power (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hoover Dam</td>
<td>$1.34 \times 10^9$</td>
</tr>
<tr>
<td>Car moving at 40 mph</td>
<td>$7 \times 10^4$</td>
</tr>
<tr>
<td>Home stove</td>
<td>$1.2 \times 10^4$</td>
</tr>
<tr>
<td>Sunlight falling on one square meter</td>
<td>1380</td>
</tr>
<tr>
<td>Refrigerator</td>
<td>615</td>
</tr>
<tr>
<td>Television</td>
<td>200</td>
</tr>
<tr>
<td>Person walking up stairs</td>
<td>150</td>
</tr>
<tr>
<td>Human brain</td>
<td>20</td>
</tr>
</tbody>
</table>
7-4 Power

If an object is moving at a constant speed in the face of friction, gravity, air resistance, and so forth, the power exerted by the driving force can be written:

\[ P = \frac{Fd}{t} = F\left(\frac{d}{t}\right) = Fv \quad (7-13) \]
Summary of Chapter 7

• If the force is constant and parallel to the displacement, work is force times distance

• If the force is not parallel to the displacement,

\[ W = (F \cos \theta)d = Fd \cos \theta \]

• The total work is the work done by the net force:

\[ W_{\text{total}} = (F_{\text{total}} \cos \theta)d = F_{\text{total}}d \cos \theta \]
Summary of Chapter 7

- SI unit of work: the joule, J
- Total work is equal to the change in kinetic energy:

\[ W_{\text{total}} = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \]

where

\[ K = \frac{1}{2}mv^2 \]

Kinetic energy is either positive or 0, never negative.
Summary of Chapter 7

• Work done by a spring force:

\[ W = \frac{1}{2} kx^2 \]

• Power is the rate at which work is done:

\[ P = \frac{W}{t} \]

• SI unit of power: the watt, W
Review, previously:

\[ W = F \cdot d \cos \theta \]

1. \[ W = I F \cdot l d \cos \theta \]

\[ \theta = 0 \quad W = F \cdot d = |F| \cdot |d| \]

\[ \theta = 90^\circ \quad W = 0 \quad \cos 90^\circ = 0 \]

\[ \theta = 180^\circ \quad W = -|F| \cdot |d| \]

\[ \text{Example: Normal force} \]

\[ \text{Example: Kinetic friction} \]

\[ d = \text{distance positive} \]

F, here use its size. (positive)

Check \( \cos \theta \) to determine the sign of \( W \).

\[ \begin{cases} \text{if:} & 0 < \theta < 90^\circ, \cos \theta \geq 0, \quad W \geq 0. \\ \text{if:} & 90^\circ < \theta \leq 180^\circ, \cos \theta \leq 0, \quad W \leq 0 \end{cases} \]

\( F \) and \( d \) need to co-exist (positive)

Hit a ball with \( F \) ball moves \( F \cdot L \),

No, \( W = F \cdot d \), \( d < L \)

Q: Throw a ball upward. How to know the amount of work?

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\[ \text{Kinetic Energy: } K \text{ or KE} \]
\[ K = \frac{1}{2} m v^2 \]

\[ \Delta K = \frac{1}{2} m (v_f^2 - v_0^2) = W_{\text{Total}} \]

Total work done on object \( > 0 \), kinetic energy \( \uparrow \)
Total work \( < 0 \), kinetic energy \( \downarrow \)
Total work \( = 0 \), kinetic energy \( \text{no change} \)

1. To stop a car. \( v_0, m, v_f = 0 \)
\[ \Delta K = 0 - \frac{1}{2} m v_0^2 = W_F = -F \cdot d \]
\[ \frac{1}{2} m v_0^2 = F \cdot d \]

If the force of resistance is fixed, double the mass. \( d \) will double. Double the \( v_0 \), \( d \) will become 4 times.