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**Problem 1. Practice with proportions.** Two masses a distance $R$ apart attract each other with a gravitational force $F$. For each of the cases below, give the factor by which $F$ is changed, if:

a.) if $R$ is multiplied by 3, then $F_{new} = \frac{F_{old}}{3}$

b.) if $R$ is multiplied by 3.9, then $F_{new} = \frac{F_{old}}{3.9}$

c.) if $R$ is divided by 2, then $F_{new} = \frac{F_{old}}{2}$

**Solution.** The equation that we are analyzing is $F = \frac{Gm_1m_2}{r^2}$.

We can see that it is proportional to $1/r^2$ - that is to say, the equation divides by the square of $R$. So, if $R$ is multiplied by a certain factor, the force is divided by the square of that factor. Or, equivalently, the force is multiplied by $(1/(\text{the factor}))^2$.

(a) The factor is 3. So $F_{\text{new}} = F_{\text{old}} \left(\frac{1}{3}\right)^2 = F_{\text{old}} \times 0.111111$

(b) The factor is 3.9. So $F_{\text{new}} = F_{\text{old}} \left(\frac{1}{3.9}\right)^2 = F_{\text{old}} \times 0.0657$

(c) The factor is 0.5. So $F_{\text{new}} = F_{\text{old}} \left(\frac{1}{0.5}\right)^2 = F_{\text{old}} \times 4$

Never forget the power of square. 10 inch Pizza = FOUR 5 inch pizza. To reduce the radius to half, you reduce the area to $\frac{1}{4}$. To increase the distance by a factor of two, you reduced the gravity force by a factor of 4.

**Problem 2.** Mass A (5 kg) and mass B (4 kg) sit on the ground, 5 m apart.

a.) What is size of the gravitational force:

- on A from B? \[ F_{\text{A\rightarrow B}} = \] \[ N \]
- on B from A? \[ F_{\text{B\rightarrow A}} = \] \[ N \]

b.) What is the size of the gravitational force:

- on A from the Earth? \[ F_{\text{A\rightarrow Earth}} = \] \[ N \]
- on B from the Earth? \[ F_{\text{B\rightarrow Earth}} = \] \[ N \]

c.) Explain why we normally only worry about the gravitational force on such objects from the Earth, and don't worry about including the gravitational force on objects from each other.

d.) Do A and B exert forces back on the Earth itself? If so, are those forces of smaller, equal, or larger size than your answers for part b?

\[ \text{same size} \]

In this problem, be especially wary of round-off; keep 2-3 (non-zero) digits in each answer and throughout your calculations.

**Solution.** (a) There is just one force to calculate, since these are Newton's-Third-Law paired forces, equal and opposite. A attracts B with the same amount of force as B attracts A

\[ F = \frac{Gm_1m_2}{r^2} = \frac{(6.67 \times 10^{-11} \text{ N-kg}^2/\text{m}^2)(5 \text{ kg})(4 \text{ kg})}{(5 \text{ m})^2} = 5.336 \times 10^{-11} \text{ N} \]

(b) Now put in Earth's mass in place of $m_B$ and Earth's radius in place of the 5-m distance.

\[ F_B = \frac{GM_1m_B}{r^2} = \frac{(6.67 \times 10^{-11} \text{ N-kg}^2/\text{m}^2)(5.97 \times 10^{24} \text{ kg})(4 \text{ kg})}{(6.37 \times 10^6 \text{ m})^2} = 39.25 \text{ N} \]
Oops! ☺ That was sure silly! I should have used our old friend \( g = 9.8 \text{ N/kg} \)

\[
\begin{align*}
F_A &= m_A g = (5 \text{ kg})(9.8 \text{ N/kg}) = 49 \text{ N} \\
F_B &= m_B g = (4 \text{ kg})(9.8 \text{ N/kg}) = 39.2 \text{ N}
\end{align*}
\]

(At least the complicated way gets the same answer.) That's simply because \( GM_E/R_E^2 = 9.8 \text{ N/kg} \).

**Problem 4.** Based on the following data about planet X (which orbits around the Sun):
Planet X’s distance from Sun = \( 3 \times 10^{12} \) m
Planet X’s radius = \( 1.2 \times 10^6 \) m
Planet X’s mass = \( 7 \times 10^{22} \) kg

a.) Find \( g_x \), the size of the acceleration due to gravity on the surface of Planet X. \( \text{m/s}^2 \)

b.) What is the weight of a 10 kg mass on the surface of Planet X? \( \text{N} \)
(How does this compare to its weight on Earth?)

c.) How long would it take for a ball dropped from a height of 12 m to hit the ground? \( \text{s} \)
(How does this compare to the time it would take on Earth?)

d.) At 1 of Planet X’s radii above the planet’s surface, what is \( g \)? \( \text{m/s}^2 \)

e.) Find the orbital speed of Planet X around the Sun. \( \text{m/s} \)

f.) How long is a year on Planet X? Express your answers in both seconds and Earth years: \( \text{s} \) \( \text{Earth years} \)

**Solution.** (a) Object \( m_B \) on the surface of planet X will be attracted by planet X by

\[
F = \frac{GM_x m_B}{r_x^2}
\]

This gravity force will cause object \( m_B \) to have acceleration: 

\[
g_x = \frac{F}{m_B} = \frac{GM_x}{r_x^2}
\]

\[
g_x = \frac{GM_x}{R_x^2} = \left(\frac{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2}{(7 \times 10^{22} \text{ kg})}\right) \left(1.2 \times 10^6 \text{ m}\right)^2 = 3.242 \text{ N/kg}
\]

Notice that you should use the radius of planet here, because \( r \) is the distance between mass center of the object and the planet, and the object is close to the planet surface.

(b) Use the usual expression for weight,

\[
F = \frac{GM_x m_B}{r_x^2} = g m_B
\]

\[
10 \times 3.24 = 32.4 \text{ Newton}
\]

(c) The time to go a distance \( x \), starting from rest, with acceleration \( g \), is obtained from

\[
x = \frac{1}{2} gt^2,
\]

giving

\[
t = \sqrt{\frac{2x}{g}} = \sqrt{\frac{2(12 \text{ m})}{3.242 \text{ m/s}^2}} = 2.721 \text{ s}
\]

(On Earth, \( t = \sqrt{\frac{2x}{g}} = \sqrt{\frac{2(12 \text{ m})}{9.8 \text{ m/s}^2}} = 1.565 \text{ s} \). A shorter time, since gravity on earth is stronger.)

(d) This means that the distance from the center of planet X is doubled. \( g \) decreases proportionally to the inverse of the distance, squared. So it will go down by a factor of 1/4, to

\[
g_{1 \text{ radius up}} = \frac{g_{\text{surface}}}{4} = \frac{(3.242 \text{ N/kg})}{4} = 0.811 \text{ N/kg}
\]
(e) and (f) Don't try to memorize any conclusions from chapter 12. All you need to know is that 
\[ F = \frac{Gm_1m_2}{R^2} \] This is the only force acting on planet to keep the planet in orbit. It's pointing to the center.

And for circular orbit we know that the total force in the radius direction pointing to the center is:
\[ \sum F_r = \frac{m_2v^2}{R} \]
All we need to do for circular orbit is to solve the equation:
\[ F = \frac{Gm_1m_2}{R^2} = \frac{m_2v^2}{R} \]
\[ m_1 \text{ is the mass of the Sun and } m_2 \text{ is the mass of the planet, } v \text{ the orbit velocity of planet, and most importantly here } R \text{ is the distance between sun and the planet. Not the radius of the planet or radius of the sun!} \]

From this equation we see, \( v^2 = \frac{Gm_1}{R} \), we can solve that \( v = \sqrt{\frac{Gm_1}{R}} \), \( R = 3.0 \times 10^{12} \text{ m} \) is the distance between the sun and the planet, \( m_1 = 2.0 \times 10^{30} \), Hence, \( v = 6.77 \times 10^3 \text{ m/s} \)
Notice that the further the planet is from the sun, the slower \( v \) will be.

The total circle length is \( 2\pi R \), and the total time to finish one circle is \( T = 2\pi R/v = 2.82 \times 10^9 \text{ s} \) or \( 2.82 \times 10^9 \text{ s} \)
It is \( 2.82 \times 10^9 / (365 \times 24 \times 3600) = 89.44 \text{ years} \). Also notice that \( T = 2\pi R/v \), hence,
\[ T = \frac{2\pi R}{\sqrt{Gm_1}} \]
the further the planet is from the sun, the longer one solar year will be.

You need not to memorize any of these facts, \( \frac{Gm_1m_2}{R^2} \) solves all items for you.
DO NOT USE THE WRONG \( R \).

**Problem 4 = Walker P 7-08.** A child pulls a friend in a little red wagon with constant speed. If the child pulls with a force of 16 N for 14.4 m, and the handle of the wagon is inclined at an angle of 28° above the horizontal, how much work does the child do on the wagon?

**Solution.** "Work = force times distance." This means the component of the force in the direction of the motion:
\[ F_{\text{parallel to motion}} = f \cos \theta = (16 \text{ N}) \cos 28^\circ = 14.13 \text{ N} \]
Then the work is equal to 
\[ W = F_{\text{parallel to motion}} d = (14.13 \text{ N})(14.4 \text{ m}) = 203.4 \text{ J} \]

**Problem 5.** A man lifts a 3 kg crate from the floor to a shelf (5 m high) at constant speed. (Neglect the very start and very end of the lift, where the crate would of course need to be accelerated from/to rest.)

a.) What is the work done by the man on the crate
b.) What is the work done by gravity on the crate?
c.) What is the net work done on the crate
d.) Since the crate is lifted at constant speed, what should the change in its KE be? Is this confirmed by part c?
Solution.  (a) The man pushes (he exerts the normal force) in the direction of motion:
\[
W_{\text{by man on crate}} = Fd = mgd = (3 \text{ kg})(9.8 \text{ m/s}^2)(5 \text{ m}) = 147 \text{ J}
\]
(b) The force of gravity works in the opposite direction, so the work is equal in magnitude but negative:
\[
W_{\text{by gravity on crate}} = -147 \text{ J}
\]
(c) The net work is the sum of the two previous amounts calculated, or \( W_{\text{net}} = 0 \)
(d) Since the velocity does not change, the kinetic energy does not change. And this agrees with the fact that no net work was done on the crate.

Problem 6, A mass is attached to the bottom of a vertical spring. This causes the spring to stretch and the mass to move downward.
(a) Does the potential energy of the spring increase, decrease, or stay the same during this process?
- [ ] increase

The string will be stretched longer by amount of \( x \), so that the spring potential energy (elastic energy) of the spring increases. \( \frac{1}{2} k x^2 \), That much energy is stored in the string.
(b) Does the gravitational potential energy of the mass increase, decrease, or stay the same during this process?
- [ ] decrease

The object is lowered down by distance \( x \), the gravitational potential decreases by \( mgx \).

The higher it is, the larger the gravitational potential.

Problem 7. You and a friend both solve a problem involving a skier going down a slope. When comparing solutions, you notice that your choice for the \( y=0 \) level is different than the \( y=0 \) level chosen by your friend.
Will your answers agree or disagree on the following quantities?
(a) The skier's potential energy

The exact value of gravitational potential energy depends on the zero ground point you chose.
\( PE = mgy \), For the same object at the same position, if \( y=0 \) level is chosen differently, \( y \) is different so that PE is different. If you choose the initial position of the object to be \( y=0 \), its PE=0.
If you choose some lower points to be \( y=0 \), its PE>0.  

(b) The skier's change in potential energy

The change of the potential energy is \( mgY_{\text{final}} - mgY_{\text{initial}} = mg \text{ *(change of height)} \).
No matter the ground \( y=0 \) level was chosen differently or not the relative change of height will always be the same. For example the object lowered the location by 5 meters. Change of the PE is only determined by change of height.
No matter PE went from 0 to -100J
Or from 150 J to 50 J, the change are both 50 J.

(b) The skier's kinetic energy

The change of Kinetic energy comes from the lose of potential energy .
No matter where you choose to calculate as the zero ground, the change of height, change of PE will be the same and the increase of KE will be the same.