• Note that we have a general result for the potential at distance \( r \) from a point source charge \( Q \):

\[
V = \frac{kQ}{r}
\]

• Note also that potential is a scalar quantity

• Note that if we were at distance \( h \) above the surface of a sphere of radius \( R \) (\( R >> h \)), filled uniformly with negative charge \(-Q\), the potential would be

\[
V = -\frac{kQ}{(R + h)} \cong -\frac{kQ}{R^2} (h - R) = E[R]h + \text{Const}
\]

• Thus the electric potential energy of a charge \( q \) at height \( h \) would be

\[
U_e = qEh + \text{const}
\]

• Compare gravitational potential energy of mass \( m \) at height \( h \) above surface of Earth (large sphere filled uniformly with mass):

\[
U_g = mgh + \text{const}
\]

• Electron-volt energy unit: If an electron charge (1.6 x 10\(^{-19}\)C) "moves through" (changes) electric potential by 1 V, the change in potential energy is 1 electron-volt (eV). [1 eV = 1.6 x 10\(^{-19}\) J]

### Calculation of \( E \) from \( V \)

• We saw how to get \( V \) as an integral of \( \mathbf{E} \cdot d\mathbf{r} \); can also get \( \mathbf{E} \) as a (vector) derivative of \( V \):

\[
\mathbf{E} = \frac{\partial}{\partial x} \mathbf{i} + \frac{\partial}{\partial y} \mathbf{j} + \frac{\partial}{\partial z} \mathbf{k}
\]

• Simple cases: \( V \) only changes in \( x \)-direction:

\[
\mathbf{E} = \frac{dV}{dx} \mathbf{i}
\]

• Note: \( V/m \) is also a unit for \( E \); same size as N/C

• Question: If \( V = (3 \text{ V/m})x \), what is the electric field strength \( E \) at \( x = 2 \text{ m} \)?

### Potential from a Set of Point Charges

• We can get the potential at point \( P \) from a set of charges \( Q_i \) at distance \( r_1 \), \( Q_2 \) at \( r_2 \), … \( Q_i \) at \( r_i \) by superposition. Since \( V \) is scalar, the effects just add:

\[
V[P] = \sum_i V_i[P] = \sum_i \frac{kQ_i}{r_i}
\]

• Question: What is potential at origin if \( kQ_1 = kQ_2 = kQ_3 = 3 \text{ (Vm)} \)

### Potential from Continuous Charge (Integration)

• For a line of charge with linear charge density \( \lambda \),

\[
dQ = \lambda dx.
\]

If \( d \) is the distance of \( dQ \) from the observation point \( P \), then

\[
V = \int \frac{kdQ}{r}
\]

• Example

\[
\lambda = Q/l \quad \text{dQ} = \lambda dx
\]

Distance from \( P \) is \( a + l - x \)

\[
\Delta y \quad \text{distance from P is} \quad \Delta x
\]

\[
\text{Question: If P is distance z above origin, what is potential at P?}
\]

\[
dV = \frac{k\lambda dx}{a + l - x}
\]

\[
V = \int_{x=0}^{x=l} \frac{k\lambda dx}{a + l - x} = -k\lambda \left[ \ln(a + l - x) \right]_0^l
\]

\[
= -k\lambda \left( \ln(a) - \ln(a + l) \right)
\]

\[
= k\lambda \ln \left( \frac{a + l}{a} \right)
\]
Potential from a Disk of Charge

Circular filament (we already know the potential at $P$ from it!)

\[ \sigma = \frac{Q}{\pi a^2} \]

Integrate over set of rings, each of radius $r$ and thickness $dr$, with charge $dQ = \sigma 2\pi r dr$

Potential Energy of System of Charges

Total $U =$ Sum over (Distinct) Pairs

\[ U = \sum_{Pairs} \frac{kQ_iQ_j}{r_{ij}} = \frac{1}{2} \sum_i Q_i V_i \quad V_i \text{ from other charges} \]

- Question: If $kQ_iQ_j = 2 \text{ Nm}^2$ for each charge pair, what is the electric potential energy of system?