Ex. 2: Integration - Disk of charge

Circular filament (we already know the field at P from it!)
Field is along \( \hat{k} \)

\[ \sigma = \frac{Q}{\pi a^2} \]

Integrate over set of rings, each of radius \( r \) and thickness \( dr \), with charge \( dQ = \sigma 2\pi r dr \)

\[
\hat{E} = k\sigma \pi z k \int_{r=0}^{a} \frac{2rdr}{(z^2 + r^2)^{3/2}}
\]

\[
\hat{E} = 2k\sigma \pi k \left( 1 - \frac{z}{\sqrt{z^2 + a^2}} \right)
\]

Volume charge density \( \rho \) and Gauss's law

- Volume charge density \( \rho \): \( dQ = \rho dV \);
- If volume charge density is uniform, \( \rho = Q/V \);
- Example: Infinite "slab" of charge (Gauss's law)

Uniform volume charge density \( \rho \)

Field above slab same as that of infinite sheet of charge with \( \sigma = \rho w \)

\[
\hat{E} = \frac{\rho w}{2\varepsilon_0} \hat{k}
\]

Inside slab, use Gaussian box going from -z to +z with area \( A \) top and bottom. Again, \( \hat{E} = E \hat{k} \), so

\[
\Phi_E = EA + EA = \frac{Q_{im}}{\varepsilon_0} = \frac{\rho (2z) A}{\varepsilon_0}
\]

\[
2EA = \frac{2\rho zA}{\varepsilon_0}
\]

\[
E = \frac{\rho z}{\varepsilon_0}
\]

\[
\hat{E} = \frac{\rho z}{\varepsilon_0} \hat{k} \ldots \text{ for } |z| < \frac{w}{2}
\]

Note that the sign automatically changes for \( z < 0 \)
Question: Start at midpoint and move up; how does \( E \) change?

Motion of charge in Electric Field

- Nothing new - electric force acts like any other force; charge \( q \) of mass \( m \) in field \( E \) has acceleration:

\[
\vec{a} = \frac{q}{m} \vec{E}
\]

- Object containing equal amounts of pos. & neg. charge will not feel force in uniform \( \vec{E} \), but will feel force if \( \vec{E} \) varies with position.
Electric Dipole

- Common arrangement of charges:
  \[ -Q \quad +Q \]
  \[ \overrightarrow{p} \]
- No net charge on dipole, but charges separated
- Example: water molecule
- Dipole moment \( \overrightarrow{p} = Q \overrightarrow{l} \) where \( \overrightarrow{l} \) is the vector from the negative charge to the positive charge

Forces and Torques on Dipole

- Dipole placed in uniform field feels no net force
  \[ \overrightarrow{F}_{\text{total}} = Q\overrightarrow{E} + (-Q)\overrightarrow{E} = 0 \]
- Feels net torque:
  \[ \overrightarrow{\tau}_{\text{total}} = \sum_{i} \overrightarrow{r}_{i} \times \overrightarrow{F}_{i} \]
  \[ \overrightarrow{r}_{i} = 0 \ldots \overrightarrow{r}_{3} = \overrightarrow{l} \]
  \[ \overrightarrow{\tau}_{\text{total}} = \overrightarrow{l} \times Q\overrightarrow{E} = Q\overrightarrow{l} \times \overrightarrow{E} \]
Thus torque on dipole is
  \[ \overrightarrow{\tau} = \overrightarrow{p} \times \overrightarrow{E} \]

Question: Direction of torque?

Torque example

- What is the magnitude of the torque on a dipole with \( p = 0.01 \text{ C} \cdot \text{m} \) and in field of strength \( E = 100 \text{ N/C} \) if the angle between \( \overrightarrow{p} \) and \( \overrightarrow{E} \) is 30°?
- Is the torque on a dipole in an E field ever zero?

Electric Potential Energy

- Remember gravitational potential energy \( U = mgh \) for object of mass \( m \) at height \( h \) above zero potential reference position (valid near Earth surface)
  - Describes energy stored when work done against conservative force (gravity)
  - \( g \) is strength of Earth gravitational field (in N/kg)
  - We must do work on mass \( m \) to increase its gravitational potential energy
- A charged particle in an electric field has electric potential energy (electric force is conservative)

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<tr>
<td>Charge</td>
<td>Electric</td>
<td>Electric</td>
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