Electric Field $\vec{E}$

- $\vec{E}$ is a disturbance in space around a charge
- $\vec{E}[\vec{r}]$ is the electric field vector at position $\vec{r}$
- $\vec{E}[\vec{r}]$ is the electric force that would act on a charge $q$ placed at $\vec{r}$, divided by $q$:

$$\vec{E}[\vec{r}] = \frac{\vec{F}_q}{q}$$

- Unit of $E$: newton/coulomb (N/C)
- (Alternate unit volts/meter (V/m))
- $q$ sometimes called *test charge*
- All other charges (sources) held fixed
Electric Field of Point Charge Q (at origin)

- If we put “test” charge q at position \( \vec{r} \), Coulomb’s law says:
  \[
  \vec{F}_{qQ} = \frac{kQq\hat{r}}{r^2}
  \]

- Thus the electric field at position \( \vec{r} \) due to charge Q at the origin is:
  \[
  \vec{E}[\vec{r}] = \frac{kQ\hat{r}}{r^2}
  \]

- Sign of Q matters: \( \vec{E} \) points away from + charge, toward - charge.

- A charge q placed at position \( \vec{r} \) in E field feels force
  \[
  \vec{F}_q = q\vec{E}[\vec{r}]
  \]
Vector Diagram of \( \vec{E} \) for point charge
2D Vector Diagram for $\vec{E}$ from point charge
Example: Calculation of E from point charge

• Source charge \( Q = -1.0 \times 10^{-6} \, \text{C} \) at origin; find electric field at \( \vec{r} = -1 \, \text{m} \, \hat{i} - 2 \, \text{m} \, \hat{j} \)
Superposition of Electric Fields

- Field at \( r \) produced by several source charges is VECTOR sum of fields from each charge:

\[
\vec{E}_{total}[\vec{r}] = \sum_i \vec{E}_i[\vec{r}] = \sum_i \frac{kQ_i}{r_i^2} \hat{r}_i
\]

- Example:

\( Q_1 = +10 \ \mu \text{C} \) at origin

\( Q_2 = -5 \ \mu \text{C} \) on x-axis at \( x = 2 \text{m} \)

Find total electric field at \( \vec{r} = 2\text{m} \hat{i} + 2\text{m} \hat{j} \)
Field Lines and Field Line Map

- Vector map: set of arrows at discrete locations; length of arrow proportional to field strength, direction of arrow gives field direction

- Field Line Map: Continuous lines; density of lines proportional to field strength, tangent to line gives field direction at that point
Drawing field lines:

1. Field lines come out of + charge; go into - charge
2. # field lines entering or leaving charge is proportional to amount of charge
3. Field lines spherically symmetric near point charge
4. If system has net charge, field lines are spherically symmetric at locations far from all charges
5. Field lines never cross (tend to “repel” each other)
6. Field lines are smooth
7. For closed volume: if number of field lines exiting larger than number entering, net + charge inside
Field line map of pair of + and - charges