Calculation of E as Function of Position

- Last time, we calculated E at one specific location. Often we also want E as a function of position.
- \( E_1 = E_2 = kQ/(a^2 + y^2) \)
- \( (E_1)_x = -(E_2)_x \)
- \( (E_1)_y = (E_2)_y = E_1 \cos \theta \)
- \( \cos \theta = \frac{y}{\sqrt{a^2 + y^2}} \)
- \( \vec{E} = 2E_1 \cos \theta \hat{j} \)
- \( = \frac{2kQy}{(a^2 + y^2)^{1.5}} \)

Field lines vs. net charge in a volume

- Net # of electric field lines emerging from a volume is proportional to net charge inside volume:
  - Positive # net field lines emerge: \( Q_{\text{in}} \) positive
  - Negative # net field lines emerge (i.e., more lines enter than leave): \( Q_{\text{in}} \) negative
  - Zero net field lines emerge: \( Q_{\text{in}} = 0 \)
  (There might be charges inside, but total \( Q_{\text{in}} = 0 \))
- This is a crude form of Gauss's Law
- **Question**——

Electric Flux \( \Phi_E \)

- \( \Phi_E \) is the amount of “flow” of \( \vec{E} \) through a surface
- For a small element \( dA \) of area with normal \( \hat{n} \) (a small enough \( dA \) that \( \vec{E} \) is same everywhere on it)
  \[ d\Phi_E = (\vec{E} \cdot \hat{n})dA = E(\cos \theta)dA \]
- \( \theta \) is the angle between \( \vec{E} \) and \( \hat{n} \)
- \( \hat{n} \) is taken **outward** from a closed surface
- Unit of electric flux: \( \text{N} \cdot \text{m}^2 / \text{C} \)
Electric Flux Example

- \( \mathbf{E} = (2 \mathbf{i} + 3 \mathbf{j}) \text{ N/C} \) everywhere on a surface of area 2 m\(^2\) which has the normal \((0.7 \mathbf{i} - 0.7 \mathbf{j})\). What is the electric flux through this surface?
Total electric flux out of closed surface

- **Closed surface** completely encloses a volume

\[ \Phi_E = \oint d\Phi_E = \oint (\mathbf{E} \cdot \mathbf{n}) dA \]

Flux from point charge

- What is the electric flux out of a sphere of radius \( R \) from a charge \( q \) at the center?

Gauss’s Law

- For the charge \( q \) at the center of the sphere, we found the electric flux out of the sphere = \( 4\pi k q \)
- We get the same flux for any closed surface enclosing the charge, and the charge doesn’t have to be at the center!
- Remember \( k = 1/(4\pi \varepsilon_0) \), so we could write our result as \( \Phi_E = q/k = Q_m/\varepsilon_0 \) This is Gauss’s Law:

\[ \Phi_E = \frac{Q_m}{\varepsilon_0} \]

where \( \Phi_E \) is the electric flux out of a closed surface and \( Q_m \) is the net charge enclosed by the surface.