Write all vectors in terms of standard unit vectors i, j, k, and r as needed.

1. Identical point charges of magnitude q are placed at the 3 corners of an equilateral triangle of side a. What is the magnitude of the force on any one charge due to the other 2? (Circle the best answer)
   
   A. \( 2kq^2/a^2 \)
   B. \( kq^2/a^2 \)
   C. \( 4kq^2/a^2 \)
   D. \( \frac{3\sqrt{3}}{2} \frac{kq^2}{a^2} \)
   E. \( \frac{3\sqrt{3}}{2} \frac{kq^2}{(2a)^2} \)

   \[ \begin{align*}
   \text{X-components of force cancel} \\
   \text{Y-components} & = 2 \left( \frac{kq^2}{a^2} \right) \cos \theta \\
   \cos \theta & = \frac{d}{a} = \frac{\sqrt{3}a - a}{a} = \frac{\sqrt{3}}{3} \Rightarrow F = \frac{\sqrt{3}}{3} \frac{kq^2}{a^2}
   \end{align*} \]

2. A thin sheet of charge measures 1.0 m wide by 2.0 m long. Which answer below is closest to the strength of the electric field at a position 0.010 m above the sheet of charge? (Circle the best answer)
   
   \[ Q = 0.010 \, \mu C \]

   a) 0.0025 \( (\text{C/m}^2) / \varepsilon_0 \)
   b) 0.005 \( (\text{C/m}^2) / \varepsilon_0 \)
   c) 0.010 \( (\text{C/m}^2) / \varepsilon_0 \)
   d) 0.50 \( (\text{C/m}^2) / \varepsilon_0 \)
   e) 25 \( (\text{C/m}^2) / \varepsilon_0 \)

   \[ E = \frac{Q}{2 \varepsilon_0} = \frac{Q}{2 \varepsilon_0} \text{ from Gauss's Law (infinite sheet)} \]

3. The electric field in a region is given by \( E = (3 \, \text{V/m}^2) \hat{i} \) where \( x \) is the x-axis coordinate value. What is the value of the potential difference \( V(x=3m, y=0) - V(x=2m, y=0) \)? (Circle the best answer)
   
   \[ \Delta V = - \int_{x=2m}^{x=3m} (3 \, \text{V/m}^2) \, dx \]

   a) 15V
   b) 7.5V
   c) 3V
   d) -3V
   e) -7.5V
   f) -15V

4. The surface charge density is 2 \( \text{C/m}^2 \) and the potential is 500V at point A on a conductor. What is the potential at point B on the same conductor if the surface charge density there is 1 \( \text{C/m}^2 \)? (Circle the best answer)
   
   a) 2000V
   b) 1000V
   c) 500V
   d) 250V
   e) 125V

Conductor is an equipotential!
5. In a region of space, a cylinder of length \( L = 0.50 \) m and ends of radius \( R = 0.20 \) m. There is no flux through the sides of the cylinder. There is uniform electric field of strength \( E_1 = 4.0 \times 10^5 \) N/C entering the cylinder through one end, and through the other end there is uniform electric field of strength \( E_2 = 2.0 \times 10^5 \) N/C exiting. The total charge, in \( \mu \text{C} \), contained within the cylinder is: (Circle the best answer)

A. -0.22
B. 0.36
C. -1.4
D. +1.4
E. -0.36

\[
\Phi_E = \pi R^2 \left[ 2 \times 10^5 \text{N/C} - 4 \times 10^5 \text{N/C} \right] = \frac{Q_{in}}{E_0}
\]

\[
Q_{in} = \pi E_0 (0.2 \text{m})^2 (2 \times 10^5 \text{N/C}) = -2.22 \times 10^{-7} \text{C}
\]

\[
= -0.22 \mu \text{C}
\]

6. Charge is uniformly distributed throughout a long cylinder of radius 0.05 m. At a point a distance of 0.10 m from the center of the cylinder the electric field is \( 8.9 \times 10^5 \) N/C. The field, in N/C, at a distance of 0.04 m from the center of the cylinder is: (Circle the best answer)

A. \( 3.5 \times 10^3 \)
B. \( 5.5 \times 10^5 \)
C. \( 1.4 \times 10^5 \)
D. \( 3.4 \times 10^3 \)
E. \( 1.4 \times 10^6 \)

\[
\text{By Gaussian's Law: } 2 \pi R L E_1 = \frac{Q_{in}}{E_0} \Rightarrow E_2 = \frac{2 \pi R L E_1}{E_0} = \frac{2 \pi (0.05 \text{m}) (0.1 \text{m}) \times 8.9 \times 10^5 \text{N/C}}{0.04 \text{m}}
\]

\[
= 1.4 \times 10^6 \text{N/C}
\]

7. A charge of 1.3 \( \mu \text{C} \) is uniformly distributed along the x-axis from \( x = -0.10 \) m to \( x = +0.10 \) m. The potential, in V, at the point \( x = 0.20 \) m is: (Circle the best answer)

A. \( 0.8 \times 10^4 \)
B. \( 8.5 \times 10^4 \)
C. \( 9.2 \times 10^4 \)
D. \( 8.5 \times 10^4 \)
E. \( 6.4 \times 10^4 \)

\[
V = \int_{-0.1 \text{m}}^{+0.1 \text{m}} \frac{k(Q/2)}{(0.2 \text{m} - x)} dx
\]

\[
= -kQ \ln(0.2 \text{m} - x) \bigg|_{-0.1 \text{m}}^{+0.1 \text{m}}
\]

8. A spherical conductor has a radius of \( R = 0.15 \) m. The potential at a distance of 0.45 m from the center of the sphere is measured to be \( 2.50 \times 10^2 \) V. The surface charge density, in C/m\(^2\), on the sphere is: (Circle the best answer)

A. \( 4.4 \times 10^8 \)
B. \( 5.6 \times 10^8 \)
C. \( 5.6 \times 10^8 \)
D. \( 3.5 \times 10^8 \)
E. \( 3.5 \times 10^8 \)

\[
V = \frac{kQ}{4 \pi R^2} \Rightarrow Q = \frac{4 \pi R^2 V}{k}
\]

\[
= \frac{4 \times 2.5 \times 10^{-8}}{4 \pi (0.15 \text{m})^2} = 4.4 \times 10^{-8} \text{C/m}^2
\]

Longer Problems. Show all work, and derive any needed relations from the "Starting Equations."

1. A conductor is in the shape of a spherical shell with inner radius \( R_1 \) and outer radius \( R_2 \). There is no net charge on the conductor. At the center of the spherical shell is a point charge \( +Q \).

(a) Draw a set of field lines on the diagram
(b) If $r$ is the radial distance from the center, write down the electric field in the region $0 < r < R_1$ in terms of $Q$, $r$, $R_1$, $R_2$, the electric constants $k$ and $\varepsilon_0$, and appropriate unit vectors and standard constants as needed.

$$\frac{4\pi}{r^2} E = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi \varepsilon_0 r^2}$$

(c) Write down the electric field in the region $R_1 < r < R_2$

$$0 \quad \text{(conductor)}$$

(d) Write down the electric field in the region $r > R_2$

$$\frac{4\pi}{r^2} E = \frac{Q}{\varepsilon_0} \implies E = \frac{Q}{4\pi \varepsilon_0 r^2}$$

2. Three charges are arranged as follows: A charge of 2C is at $(x,y) = (0, 1m)$; a charge of 2C is at $(1m, 0)$; and a charge -2C is at $(0, -1m)$. What is the electric field at the origin?

$$G_2 = 2C$$

$$2C = Q_2$$

$$0 \quad -2C = Q_3$$

$$E_x = E_2 = \frac{k Q_2}{(1m)^2} = -1.8 \times 10^{10} \text{ N/C}$$

$$E_y = E_1 + E_2 = -2 \left( \frac{k Q_1}{(1m)^2} \right) = -3.6 \times 10^{10} \text{ N/C}$$

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{(-1.8 \times 10^{10})^2 + (-3.6 \times 10^{10})^2} = 4.2 \times 10^{10} \text{ N/C}$$

3. A charge $Q$ is uniformly distributed on a string which lies on the x-axis between $x = -L/2$ and $x = +L/2$. Give all answers in terms of $L$, $Q$, the electric constant $k$, and other standard constants such as $\pi$, as needed.

(a) What is the linear charge density of the string?

$$\lambda = \frac{Q}{L}$$

(b) Write down the charge element $dQ$ you will use to find, by integration, the potential at position $y$ on the y-axis. Give $dQ$ in terms of $dx$, $Q$, $L$, $x$, $y$, and $k$ as needed.

$$dQ = \lambda dx$$

(c) Write down the integral for the potential at position $y$ on the y-axis, showing the upper and lower limits of integration.

$$V = \int_{-L/2}^{L/2} \frac{k Q dx}{(x^2 + y^2)^{3/2}}$$

(d) Carry out the integration to find the potential $V$. 