Capacitor Charging

\[ \Delta V_C = \frac{q}{C} \]
\[ \varepsilon - IR - \Delta V_C = 0 \]
\[ I = \frac{dq}{dt} \]
\[ R \left( \frac{dq}{dt} \right) + \frac{q}{C} = \varepsilon \]

• New differential equation. Try general exponential solution

\[ q = A + B e^{-t/RC} \]
\[ R \left( \frac{dq}{dt} \right) + \frac{A + B e^{-t/RC}}{C} = \varepsilon \]

• At \( t=\infty \) we have \( A/C = \varepsilon \), that is, \( A=C \varepsilon \)

Example

• How long after switch closed until \( \Delta V_C = 5 \text{V} \)?
• We need to find \( t \) such that

\[ (10\text{V})(1-e^{-t/RC})=5\text{V} \]

• Thus \( 1-e^{-t/RC} = 5/10 \)
• \( e^{-t/RC} = 1-0.5 = 0.5 \)
• \( -t/RC = \ln(0.5) \)
• \( t = -RC \ln(0.5) = 6.9 \times 10^{-3} \text{s} \)
• Q: What is \( \Delta V_C \) at 0.01 s after switch closed?

Inductance: Self (L) and Mutual (M)

• Magnetic flux through a circuit related to current in that circuit and current in nearby circuits:

\[ L_1 = \frac{d\Phi_{B1}}{dt} = L_1 I_1 + M_{12} I_2 \]

\[ \Phi_{B1} = L_1 I_1 + M_{12} I_2 \]

Self Inductance of Circuit 1

• \( L_1 \) depends only on geometry of circuit 1
• \( M_{12} \) depends on geometry of both circuits \((M_{12}=M_{21}=M)\)
• If one or both of the currents change, get induced emf's

\[ \varepsilon_1 = -\frac{d\Phi_{B1}}{dt} = -L_1 \frac{dI_1}{dt} - M \frac{dI_2}{dt} \]

\[ \varepsilon_2 = -\frac{d\Phi_{B2}}{dt} = -L_2 \frac{dI_2}{dt} - M \frac{dI_1}{dt} \]

• Unit of L & M is Henry (H); 1H= 1 T m^2/A = 1V/(A/s)
• For now, focus on self inductance L of one circuit
Example - Inductance of solenoid

- Solenoid with N turns, length l, current I, area enclosed by each turn is A
  - Inside B = \(\mu_0 NI/l\)
  - Outside B = 0
  - Area “enclosed” by solenoid = NA

\[
\Phi_B = \oint B \cdot ndA = BNA = \mu_0 \frac{NI}{l} NA
\]

\[
L = \frac{\Phi_B}{I} = \mu_0 N^2 A
\]

- Q: L of solenoid with l=0.1m, A=0.2 m², N=100

Example 2: Mutual Inductance

- Mutual inductance between long wire & loop

\[
d\Phi_B = \frac{\mu_0 I}{2\pi} dx
\]

\[
\Phi_B = \frac{\mu_0 cI}{2\pi} \int_{x=a}^{b} dx = \frac{\mu_0 cI}{2\pi} \left[ \ln(b) - \ln(a) \right]
\]

\[M = \frac{\Phi_B}{I} = \frac{\mu_0 c}{2\pi} \ln\left(\frac{b}{a}\right)\]

Inductance & Induced emf

- Book definition of self inductance L

\[
L = \left| \frac{d\Phi_{ind}}{dI} \right|
\]

- Self Inductance (L) : The self-inductance of a circuit element (piece of conductor) tells how much “back” emf is generated when current changes

- Circuit symbol: \[\begin{array}{c}
\text{L}
\end{array}\]

- Any circuit has self inductance. Coil or solenoid with many turns has large inductance; called an inductor. We usually neglect small inductances of straight wires.

Self-Inductance of Coax Cable

- Length l of coax cable; inside wire radius a, cylindrical shell radius b.

\[
\Phi_B = \oint \left(\frac{\mu_0 I}{2\pi} \right) dr = \frac{\mu_0 I}{2\pi} \ln\frac{b}{a}
\]

\[
L = \frac{\Phi_B}{I} = \frac{\mu_0 I}{2\pi} \ln\left(\frac{b}{a}\right)
\]

Series and Parallel Inductors

- Inductors in series:

\[
L_s = L_1 + L_2 + \ldots + L_n
\]

- Parallel inductors

\[
\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2}
\]

- If no flux from L_1 couples to L_2 (M=0), L_s=L_1 + L_2

- Parallel inductors

\[
\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2}
\]

- If no mutual coupling