Measuring the Mass of the Earth

Introduction

In this lab, we will derive the mass of the Earth using direct measurement of the acceleration of an object at the Earth's surface and Newton's laws of motion and gravity. Sir Isaac Newton changed the way in which humankind viewed the world. His laws describing the fundamental properties of physical reality took scientists from empirical work to mathematical logic. In particular, his description of gravity gave us a means to understand how we are bound to the Earth, how the Moon is bound to the Earth, how the Earth is bound to the Sun, and so on. We now understand how one planet can perturb another or how a distant cluster of galaxies can "pull" us across immense distances.

Using Newton's formulae, the radius of the Earth, and the universal constant of gravitation (represented by the letter G), we can determine the mass of the Earth. We will be using the following equations:

1. \( F = \frac{-Gm_1m_2}{r^2} \)
2. \( F = m_2a \)
3. \( a = \frac{-2x}{t^2} \)

where \( F \) is force and \( G \) (also called “big G”) is the universal constant of gravitation \( (6.67 \times 10^{-11} \text{ m}^3/\text{kg s}^2) \). The mass “\( m_1 \)” is the mass of the Earth, and “\( m_2 \)” is the mass of an object on the surface of the Earth. “\( a \)” is the acceleration of the object, “\( x \)” is the distance the object falls, and “\( t \)” is the amount of time it takes that object to fall that distance.

We can manipulate equations 1 and 2 to get the mass of the Earth in terms of the acceleration, radius of the Earth (\( R \)), and constant of gravitation (your instructor will show you how):

4. \( m_1 = \frac{(aR^2)}{G} \)

Since we "know" \( R \) to be 6,378,000 m (let's say we measured it somehow by observing the length and angles of shadows at different places on the Earth's surface) and \( G \) was fortunately measured for us by a physicist, all we need is \( a \), the acceleration of an object at the Earth's surface.

We don't need to know the mass of the object. (Think about this for a minute: why is this true, according to the equations? Look at #4.) Although any object can be dropped, we will use an object that will experience a minimum of air resistance. We will also try to drop the object from a height that is high enough to ignore the amount of time it takes us to start and stop a stop watch, but not so high that air resistance starts to affect the results.
Procedure

Find a location suitable for dropping pennies. Measure the distance the objects will be falling (you will record this in Q2 below). Using a stopwatch, have one lab partner time how long each object takes to fall this distance. Work in groups and determine who will be dropping, who will be timing, who will be recording, who will measure the height, who has the calculator, who will be quality control, etc.

Exercise

Q1: Time how long it takes an object to drop your given distance, and repeat for at least 20 times (if you “mess up”, you may need to do this more than 20 times). Record your results in a table on your answer sheet. The table should have two columns: one for the trial number, and one for the time measured in seconds.

Q2: Measure the distance that your object fell in meters. This is the value of x in our equations.

Q3: Estimate the uncertainty in your answer to Q2, also in units of meters. This should primarily be based on the instrument and method you used to measure the distance.

Share this data with the other members of your team. For mathematical understanding, each team member should do the following calculations individually. Again, please show all calculation work on your answer sheet.

Q4: Calculate the average time taken to fall the distance by adding up all the times in your table and dividing by the number of trials.

Q5: Calculate the uncertainty in this average, using this approximate method:

1. Toss out the longest and shortest times (we are calling these “outliers”)
2. Subtract the now-shortest time from the now-longest time and divide by 2

Record your uncertainty in seconds.

Q6: Square the value of the average time of fall (take your answer from Q4 and multiply it by itself). You now have \( t^2 \) in units of sec\(^2\).

Q7: Solve for the acceleration of the object using equation 3 from the introduction. You have the value of x from Q2 and the value of \( t^2 \) from Q6, and acceleration will have units of m/s\(^2\).

Q8: The negative sign for the acceleration in equation 3 refers only to direction, so we can disregard it when determining the mass of the Earth. Solve for the mass of the Earth using equation 4 from the introduction, along with G and the radius of the Earth (also given in the introduction) and the acceleration you calculated in the previous question. Your answer will have units of kilograms (kg).
Follow-up Questions

Q9: Why is it a good idea to take many measurements and average the results?

Q10: Compare your value for the mass of the Earth to the currently accepted value: $5.97 \times 10^{24}$ kg. Are you within the same “order of magnitude” (that is, does your exponent match the currently accepted value’s exponent)?

Q11: Calculate the percentage error as follows:

\[
\left( \frac{\text{Calculated Mass} - \text{Accepted Mass}}{\text{Accepted Mass}} \right) \times 100\% = \% \text{ error}
\]

This formula gives an estimate of how far off your answer is from the accepted answer in terms of a percentage. The vertical bars mean “absolute value” (so if you end up with a negative sign after subtracting the accepted mass from your calculated mass, just ignore it).

Q12: How does your value for the acceleration from Q7 compare to the accepted value of 9.8 m/s$^2$? Repeat the percentage error calculation you did in the previous question, but using your acceleration and the accepted acceleration.

Q13: With your answers from Q12 and Q11 in mind, how does your measurement for the acceleration of an object affect your derived value for the mass of the Earth?

Q14: Given your estimates for the uncertainties in the distance and times, what is the range of values allowed for the acceleration due to gravity at the surface of the Earth? That is, what are the lowest and highest values you find given the uncertainties of your experiment? Follow a and b below to determine this range of values:

a. The highest possible value comes from combining the highest possible height with the shortest possible time, given your uncertainties. Calculate the highest value for the acceleration.

b. The lowest possible value comes from combining the shortest possible height with the longest possible time, given your uncertainties. Calculate the lowest value for the acceleration.

Q15: Does the real value for the mass of the Earth lie between your uncertainties? (You can figure this out without doing any calculations!) Explain.