Physics 121 – Spring 2013 – SFSU
Exam #1 Solutions

1.) A
Bulb #1 gets total current; other two resistors split this current between them…

2.) B
... in proportional to the resistances. \( R_2 \) and \( R_3 \) in parallel; less current goes through the more resistive choice. So \( R_2 \) gets least current.

3.) B
Examine \( P_1 = I_1^2 R_1 \): definitely not lowest, since has highest current and tied for highest resistance.

Examine \( P_2 = I_2^2 R_2 \); inconclusive since though \( I_2 \) lowest (see above), \( R_2 \) is not lowest.
(Similarly with \( P_3 \); \( R_3 \) is the lowest but \( I_3 \) is not.)

\( R_1 \) and \( R_{23} \) in series, where voltage split proportionally to resistance (higher \( R \) = higher \( V \)). Conclude that since \( R_1 > R_{23}, V_1 > V_{23}. \) So examining \( P_2 = I_2 V_2, \) find that bulb #2 has lowest current (alone) and lowest voltage (tied with \( R_3 \); in parallel with it). **Bulb 2 dissipates least power.**
[Could also look at \( P_2 = V_2^2 / R_2 \): bulb 2 has lowest \( V \) (tied with \( R_3 \)) and highest \( R \) (tied with \( R_1 \)).]

4.) D
Empty cap = short circuit when has no charge. Removes voltage difference between ends of \( R_2 \) (and \( R_3 \)) so no current flows to them momentarily-- all charge flowing through \( R_1 \) is initially redirected onto the empty cap.

5.) B
Once cap has built up some charge, has some voltage difference—current will flow onto cap and also through \( R_2 \) (and \( R_3 \)) which it is in parallel with it. When cap is full (in this circuit), it will behave like an open circuit, maintaining a constant voltage difference. (This is less than \( \varepsilon; \) current still has path to flow so voltage still drops across \( R_1 \)). With the cap no longer taking charge, circuit returns to exact currents it had before the cap was connected.

6.) C
The dielectric allows you to put more charge on the cap to get to the “full” voltage than you otherwise could have, due to the shielding of the dielectric. Once this shielding is removed, the voltage across the cap will jump up to a higher value than could otherwise have been achieved by the battery. Since it is in parallel with bulb 2 (and bulb 3), it will subject them to a higher voltage difference, meaning (see \( P = V^2 / R \)) that each will glow a little brighter. Once this excess charge is done draining through them, the cap returns to its new “full” state (not quite as full as with the dielectric) and acts as an open circuit. The resistors return to their previous state as well (as in Prob #5).

7.) B
If released from rest, a negative charge seeks higher \( V \) to get to lower PE. If it has initial PE, it wants to get rid of that. The only way to get it to lower \( V \) (and higher PE) is to give it an initial velocity (KE) in that direction. This is like “throwing a ball upward”; you can get it to momentarily go to higher PE, since it has KE to trade for that, before it falls back down to lower PE.

8.) C
A: NO; go against current through \( I_3 \). Corrected version: \( +\varepsilon_1 - I_1 R_1 + I_3 R_3 = 0 \)
B: NO; \( I_1 \) does not flow through \( R_2 \). Corrected version: \( +\varepsilon_1 - I_1 R_1 - I_2 R_2 + \varepsilon_2 - I_2 R_4 = 0 \)
C: YES. Taking a loop against current flow is just fine. \( -\varepsilon_2 + I_2 R_2 + I_3 R_3 + I_2 R_4 = 0 \)

9.) A
Current (+ charge) seeks lower \( V \). We were told the current directions were correct.
10.) C
Batteries both agree that current should flow clockwise on outside loop. They only disagree on the middle segment; the particular values for batteries (and resistances) will determine who wins.

11.) A
See homework. Closer to point charge = larger $E_{ext} = larger E_{ind}$ needed for perfect shielding = larger charge polarization halves to have access to = can create more net charge.

12.) B
Charges on spheres will have left/right polarization when placed at $P_1$ or $P_2$. In the “side-by-side” orientation, this polarization occurs across the spheres, so each individually has a net charge already (though the combo is neutral). We just need to separate the spheres in order for both to have net charge individually. We have full access to convert each half of the charge polarization into net charge for a particular sphere. (Contrast with charging by induction, where some charge must be neutralized with a ground connection in order to generate net charge.)

In the “stacked” orientation, each sphere individually will remain neutral. (And the only way to generate net charge on each is to cancel half the charge polarization by a ground connection, thus reducing the amount of net charge available for each sphere.)

13.) B
Any non-zero field will cause charge polarization. But it is the non-uniformity of the field which leads to “static cling”; the two halves of the charge polarization are pulled unequally. This leads to an attraction to the object which is the cause of that non-uniform field, hence “charged object attracts uncharged”. This is true regardless of the sign of the charged object—if you swapped the positive point charge to negative, the charge polarization would also flip, and the uncharged object still feels a leftward attraction towards the point charge.

14.) B (or C)
In the side-by-side orientation, 
Option B: simply separating the two spheres would result in each having a net charge (left one -, right one +). This would be maintained taking them far away (the charge on each would simply redistribute over the outside surface).

Option A: we can treat the two spheres as a single object and use conventional charging by induction-- cancel half the charge polarization by momentary grounding (left sphere -, right one neutral). If the spheres were then taken far away (while still touching--important), the net charge would redistribute across both, leave both with a net negative charge (then it stops being important whether they are touching or not). But each sphere won’t have as much net charge as in Option B, since the negative part of the polarization (all on the left sphere in Option B) is now shared across both.

Since there was some confusion over which of the following I meant, I will accept several answer options:
- “the orientation (side-by-side) & process (separating them) which will produce the most net charge on each sphere” (answer is B)
- “orientation (side-by-side) which can produce the most net charge, but with any process that can produce some net charge on each sphere” (answer is C; either A or B would do-- see above)

Note that this is also a catch-all if you thought you had to cover any possibility:
- “ANY orientation and process which produces net charge on each sphere” (answer is C; either A or B would do)
Answer C (covers “side-by-side”) already is the broadest possible option, so the “stacked” orientation can’t expand the palate from there. But we should note that only choice A would work for this orientation-- you need the momentarily ground connection to produce a net charge on each sphere. (The remaining negative half of the charge polarization would already be on both spheres, so unlike above, it would be irrelevant if you hold the spheres together while taking them far away; the charges do not need the ability to redistribute.)
1. \( V = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} \)

\[ = kq \left( \frac{1}{r_1} + \frac{1}{r_2} \right) \]

\[ = +6146 \text{ V} \]

b. \( q_3 \) cannot supply voltage for itself. \( q_3 \) cannot cancel \( q_1, q_2 \). 

\( r_1 = 5 \text{ m} \)
\( r_2 = 5\sqrt{2} \text{ m} = 7.07 \text{ m} \) (Pythagoras)

\( q_1 = q_2 = +2 \times 10^{-6} \text{ C} \)

\( \vec{E}_1 \) has +x comp only
\( \vec{E}_2 \) has +x, -y comps

\( \vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 \) will have components in +x, -y directions.

\( q_3 = + \), so feels \( -\vec{E} \) along \( -\vec{E} \).

\( E_{\text{nety}} = E_{1y} + E_{2y} = 0 - \frac{kq_2}{r_2^2} \sin \theta \)

\( \theta = 45^\circ \cos 45^\circ = \sqrt{2}/2 \)

\[ = \frac{(9 \times 10^9)(2 \times 10^{-6})}{25} \left( 1 + \frac{\sqrt{2}/2}{(\sqrt{2}/2)^2} \right) \]

\[ E_{\text{netx}} = +975 \text{ N/C} \]

\[ \theta = \tan^{-1} \left( \frac{E_{\text{nety}}}{E_{\text{netx}}} \right) = 15^\circ \] (cw from +x axis)

angle makes sense; small angle to +x direction since most contributions that way

\( r = 0 \), \( V = \infty \)
c.) \[ KE_f + PE_f = KE_i + PE_i \]
\[ 0 = KE_f + 0 \]
\[ KE_f = q \cdot V_i \]
\[ = (2 \times 10^{-6} \text{ C})(+6146 \text{ V}) \]
\[ = 1.23 \times 10^{-2} \text{ J} \]

\[ V_i = +6146 \text{ V} \]

\[ \text{KE}_f = \text{KE}_i \text{ (not)} \]
\[ V_{net} = 0 \text{ at r = \infty} \]

(d.)

Where E field zero?

between \( q_1 \) & \( q_2 \)

\[ \vec{E} \] opposing so \( \vec{E}_{net} = 0 \) possible

\[ E_{net} = E_1 + E_2 = 0 \] since \( q_1 = q_2 \), \( v_i = v_2 \)

\[ E_{net} = \frac{kq_1q_2}{r_1^2} + \frac{kq_1q_2}{r_2^2} = 0 \]

\[ \vec{E} \] is zero between \( q_1 \) & \( q_2 \). It occurs exactly halfway since \( q_1 = q_2 \)

as shown in diagram in part d.

d')

\[ V_1 = 7.5 \text{ m} \]

\[ V_2 = 0 \text{ m} \]

\[ q_1 = q_2 \text{ so symmetric hills (min halfway)} \]

\[ V \]

\[ +y \]

\[ +y \]

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2.)

a.)

\[ R_{12} = R_1 + R_2 = 4 \, \Omega \]
\[ \frac{1}{R_{12}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{4 \, \Omega} + \frac{1}{11 \, \Omega} = 0.27 \, \Omega^{-1} \]
\[ \frac{1}{R_{123}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{4 \, \Omega} + \frac{1}{3 \, \Omega} = \frac{1}{0.27 \, \Omega} = 3 \, \Omega^{-1} \]
\[ R_{02} = \frac{R_2 + R_{123}}{5 \, \Omega} = \frac{8 \, \Omega}{5} = 1.6 \, \Omega \]
\[ I_1 = 9.375 \, A \]
\[ I_2 = 9.375 \, A \]
\[ I_3 = 3.125 \, A \]
\[ I_4 = 12.5 \, A \]

\[ I_{\text{tot}} = I_4 = \frac{E}{R_2} = \frac{100 \, \text{V}}{8 \, \Omega} = 12.5 \, \text{A} \]

[Shortcut: note \( R_{12} = \frac{1}{3} \times R_3 \) exactly

so \( I_{12} = 3 \times I_3 \) exactly

\( I_{12} = \frac{9}{4} I_3 \)
\( I_3 = \frac{1}{4} I_{\text{tot}} = 3.125 \, \text{A} \]

[Long way:]

\[ V_4 = I_4 R_4 = (12.5 \, \text{A})(5 \, \Omega) = 62.5 \, \text{V} \]
\[ V_3 = E - V_4 = 37.5 \, \text{V} \]
\[ I_3 = \frac{V_3}{R_3} = 3.125 \, \text{A} \]
\[ I_{12} = I_{\text{tot}} - I_3 = 9.375 \, \text{A} \]

b.) \( Q \uparrow, V_c \uparrow \quad (Q = CV_c) \)

[Long way:]

\[ \tau = R_{02} C = (0.27 \, \Omega)(4 \, \text{F}) = 1.08 \, \text{s} \]
\[ Q = Q_{02}(1 - e^{-1/\tau}) = 0.25 Q_{02}(1 - 0.75) \]
\[ \text{solve for } t = -\tau \ln(0.75) = 9.21 \, \text{s} \]
\[ I = I_c e^{-t/\tau} = (0.75) I_c \]
\[ (Q \uparrow 25\% , I \uparrow 75\%) \]
\[ I_4 = 9.375 \, \text{A} \]
\[ P_4 = I_4^2 R_4 = (9.375 \, \text{A})^2 (5 \, \Omega) = 439.5 \, \text{W} \]

[Shorter way:]

\[ Q = \frac{\text{charge}}{C} = \frac{\text{current} \times \text{time}}{C} \]
\[ I = \frac{\text{charge}}{C} \frac{1}{\text{time}} = \frac{Q}{C} \frac{1}{t} \]
\[ Q = 0.25 Q_{02}(1 - 0.75) \]
\[ \text{solve for } t = -\tau \ln(0.75) = 9.21 \, \text{s} \]
\[ I = I_c e^{-t/\tau} = (0.75) I_c \]
\[ I_4 = 9.375 \, \text{A} \]
\[ P_4 = I_4^2 R_4 = (9.375 \, \text{A})^2 (5 \, \Omega) = 439.5 \, \text{W} \]

now:

\[ 75 \, \text{V (decr)} \]
\[ 25 \, \text{V (incr)} \]

Cap crowds at 25 V
now, resistors have
\[ \text{less than initially} \]

(16: next page)
c.) \( Q \propto V_c \) so \( 25\% \) hill = \( 25\% \) \( \varepsilon \) "as goes \( Q \), so goes \( V_c \)

\[ V_c = 0.25 (100V) = 25V \]

**from V schematic, OR Kirch loop:**

\[ \varepsilon = V_3 + V_Y + V_c \]

\[ 100V = V_3 + 62.5V + 25V \]

**solve for** \( V_3 = 28.125V \)

\[ V_Y = \frac{I_Y R_Y}{(9.375A) (5 \Omega)} \]

= 4.875V

\[ \text{if } V_c < \varepsilon, \text{ rest of } V \text{ drop across all resistors collectively.} \]

\[ R_Y \text{ never has it alone,} \]

**shapes:**

\( V_c \) exponential increase

(as goes \( Q \), so goes \( V_c \))

\( V_{eq} \) exponential decrease

(as goes \( I \), so goes \( V_{eq} \))

(since \( I_{eq} \) goes to 0 as cap full, all individual resistors go to \( V=0 \). Difference is only what \( V \) value started at.)

\[ V_c \]

\[ V_{eq} \]

\[ \varepsilon \]

\[ V_Y \]

\[ V_3 \]

\[ V_4 \]

\[ I_Y \]

\[ R_Y \]

\[ \text{examples:} \]

\[ \varepsilon = V_{R_h} + V_c \]

**part a (t=0):**

\[ 100V = 100V + 0V \]

\( V_4 \) **alone** was 62.5 V of that

**parts b-c: (25% \( Q_{eq} \)):**

\[ 100V = 75V + 25V \]

\( V_4 \) **alone** was 46.875 V of that