19. Temperature and Thermal Energy

So
\[ \gamma = \frac{c_p}{c_v} = \frac{7}{5} \]

19.16 \( c_v = 390 \text{ J/kg} \cdot \text{K} \)

19.17 Ideal, diatomic gas. Assume no vibrations are excited \( \Rightarrow 3 \) translational modes + 2 rotational modes = 5 modes

\[ c_v = \frac{5k}{2m} \]
\[ m = 28 \times 1.67 \times 10^{-27} \text{ kg} \]
\[ \Rightarrow c_v = \frac{(5)(1.38 \times 10^{-23} \text{ J/K})}{2 \times 28 \times 1.67 \times 10^{-27} \text{ kg}} = 740 \text{ J/kg} \cdot \text{K} \]

19.18 a) System = ball. Assume the rest of the world to be a reservoir of heat (isothermal.)
b) System = bottle, until temperature reaches 0° C then the bottle and ice until ice melts, then bottle and ice and bucket.
c) System = drill bore (gives work added) + cannon. Surroundings = water.
d) Space shuttle (atmosphere assumed isothermal — work done by gravity and air resistance).

19.19 a) Yes. Any heat entering the freezer is removed. The time scale is months to years.
b) Yes. Time scale is on the order of hours.
c) No. The body is in thermal contact with the hot sand and the cool air, and absorbs heat from the sun.
d) Yes. Minutes time scale. Air has low thermal conductivity.

19.20 Put the two rooms in thermal contact. Then, by the first law, \( \Delta U = Q \), i.e., heat is exchanged until the temperatures are the same. The room at higher temperature has \( T \) \( \downarrow \) and vice versa for the other room. The hotter room has more collisions with the walls \( \rightarrow \) more balls leave the hot room than leave the cooler room. The elves in the hotter room have fewer balls to throw and the rate of throwing \( \downarrow \Rightarrow \) the \( T \downarrow \) in the hotter room. This continues until the two room rates are equal.

19.21 Absolute zero \( \rightarrow 0 \text{ K} = -273.15^\circ \text{ C} \).

\[ T \left(^\circ \text{ F} \right) = \left(-273.15^\circ \right) \frac{9}{5} + 32^\circ = -459.7^\circ \text{ F} \]

19.22 “Subzero” on the celsius scale corresponds to \( < -17.8^\circ \text{ C} \).

19.23 Mercury freezes at \(-38.87^\circ \text{ C} \).

\[ F = \frac{9}{5} C + 32^\circ \text{ F} \]

\[ T_{\text{Freeze}} = \left(\frac{9}{5}\right)(-38.87) + 32^\circ \text{ F} \]
\[ = -37.97^\circ \text{ F} \]
\[ = -5^\circ \text{ C} + 273.15 \text{ K} \]
\[ = -38.87 + 273.15 \text{ K} \]
\[ = 234.28^\circ \text{ K} \]

19.24 \( F = C = -40^\circ \text{ C} \).

19.25 By definition of the Fahrenheit scale, freezing saturated brine is at 0° F. Melting pure ice is 32° F at atmospheric pressure. Boiling water is 212° F. Assume a linear variation in length with temperature

\[ L = 1.0 \text{ cm} + \frac{10.0 \text{ cm} - 1.0 \text{ cm}}{212^\circ \text{ F} - 0^\circ \text{ F}} \times F \]

\[ = 1.0 \text{ cm} + 4.25 \times 10^{-2} \frac{\text{cm}}{^\circ \text{ F}} \times F \]

For \( F = 32 \),
\[ L = 1.0 \text{ cm} + 4.25 \times 10^{-2} \frac{\text{cm}}{^\circ \text{ F}} \times 32^\circ \text{ F} \]
\[ = 2.4 \text{ cm} \]

19.26 Kinetic energy is the same.

19.27 Two gases of different molecular masses, same temperature. Model the gas as ideal. When mixed, the resulting mixture will still have the same temperature, say \( T \). Let the masses be \( m_1 + m_2 \) with \( m_1 \geq m_2 \). Now

\[ K_1 = \frac{1}{2}m_1 \langle v^2 \rangle = \frac{3kT}{2} \]
\[ K_2 = \frac{1}{2}m_2 \langle v^2 \rangle = \frac{3kT}{2} \]

each molecule has the same kinetic energy but

\[ \langle v_1^2 \rangle = \frac{3kT}{m_1} \]
\[ \langle v_2^2 \rangle = \frac{3kT}{m_2} \]

Since \( m_1 > m_2 \) then \( \langle v_2^2 \rangle > \langle v_1^2 \rangle \).

19.28 \( \langle v_2^2 \rangle \) does not depend on pressure.

19.29 Two systems in thermal contact and in equilibrium must have the same temperature (zero-th Law) \( T \). But

\[ U = \frac{3}{2}NkT \]
which depends on $T$ and $N$. If $T$ is the same then one system can have more internal energy simply by having more particles. No, the systems do not need to have the same internal energy.

19.30 Container with 3 moles has $\frac{3}{2}$ the internal energy.

19.31 Two containers, same pressure

$$P_1 = P_2 = P$$

temperature:

$$T_1 = T_2 = T$$

but

$$V_1 = 2\, \text{L}$$
$$V_2 = 1\, \text{L}$$

We have that

$$PV = \frac{3}{2} NkT$$

and

$$U = \frac{3}{2} NkT$$

So $U = \frac{3}{2} PV$ then

$$\frac{U_1}{U_2} = \frac{\frac{3}{2} P_1 V_1}{\frac{3}{2} P_2 V_2} = \frac{V_1}{V_2} = 2$$

The 2 L container, since it contains more particles.

19.32 $P_1 + P_2 = P$, $N_1 + N_2 = N$, which gives

$$PV = NRT$$

then

$$\frac{P}{N} = \frac{P_1}{N_1} = \frac{P_2}{N_2}$$

19.33 Use

$$\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} kT$$

$$T = 20° + 273.15° = 293.15 \, \text{K}$$

$$\langle v^2 \rangle = \frac{3kT}{m}$$

$$v_{\text{rms}} = \left( \frac{3kT}{m} \right)^{\frac{1}{2}}$$

$$= \left( \frac{3 \times 1.38 \times 10^{-23} \, \text{J/K} \times 293.15 \, \text{K}}{2 \times 1.67 \times 10^{-27} \, \text{kg}} \right)^{\frac{1}{2}}$$

$$= 1910 \, \text{m/s}$$

19.34 $\rho = \frac{nm}{v}$

19.35 $T = 10^6 \, \text{K}$

a) $m_p = 1.67 \times 10^{-27} \, \text{kg}$

$$v_{\text{rms}} = \left( \frac{3kT}{m} \right)^{\frac{1}{2}}$$

$$= \left( \frac{3 \times 1.38 \times 10^{-23} \, \text{J/K} \times 10^6 \, \text{K}}{1.67 \times 10^{-27} \, \text{kg}} \right)^{\frac{1}{2}}$$

$$= 1.6 \times 10^6 \, \text{m/s}$$

b) $m_e = 9.11 \times 10^{-31} \, \text{kg}$

$$v_{\text{rms}} = \left( \frac{3 \times 1.38 \times 10^{-23} \, \text{J/K} \times 10^8 \, \text{K}}{9.11 \times 10^{-31} \, \text{kg}} \right)^{\frac{1}{2}}$$

$$= 6.7 \times 10^7 \, \text{m/s}$$

The speed of the electron is $\sim 23\%$ of the speed of light. The answer is therefore only approximate.

19.36 5.4 atm

19.37

$T_1 = 50^\circ \text{F}$

$$= (50 - 32) \frac{5}{9} + 273.15^\circ$$

$$= 283.15 \, \text{K}$$

$T_2 = 375^\circ \text{F}$

$$= (375 - 32) \frac{5}{9} + 273.15^\circ$$

$$= 464 \, \text{K}$$

For the air,

$$\frac{V_2}{V_1} = \frac{T_2}{T_1} = \frac{464 \, \text{K}}{283 \, \text{K}} = 1.64$$

The soufflé expanded by

$$\frac{V_2}{V_1} = 1.4$$

$$\frac{V_{2,s}}{V_{1,s}} = \frac{V_{\text{batter}} + V_{2,\text{air}}}{V_{\text{batter}} + V_{1,\text{air}}}$$

$$1.4 = \frac{V_{2,s}}{V_{1,s}} + 1$$

$$\Rightarrow \frac{V_2}{V_1} (1.4 - 1) = 1.64 - 1.4 = 0.24$$

$$\frac{V_2}{V_1} = \frac{0.24}{0.4} = 0.6$$

or

$$\frac{V_{\text{air}}}{V_{\text{total}}} = \frac{V_{\text{air}}}{V_{2,s} + V_{\text{air}}} = \frac{1}{V_{1,s} + 1}$$

$$= \frac{1}{1.6} = 63\%$$
19.36 We are given gauge pressure, but the gas laws are written in terms of the absolute pressure, so we convert:
\[ P = P_{\text{gauge}} + 1 \text{ atm} \]
We also have to convert the Fahrenheit temperatures to Kelvin:
\[ T = \frac{5}{9} (T \, \text{°F}) - 32 + 273 \]
So:
\[ T_1 = \frac{5}{9} (80 - 32) + 273 = 299.7 \text{ K} \]
and
\[ T_2 = \frac{5}{9} (140 - 32) + 273 = 333.0 \text{ K} \]
Then use Charles’ Law since the volume is constant:
\[ \frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow P_2 = P_1 \frac{T_2}{T_1} = (5.8 \text{ atm}) \frac{333.0 \text{ K}}{299.7 \text{ K}} = 6.44 \text{ atm} \]
Thus the gauge pressure is 5.4 atm.

19.38 In system A, no work is done since the gas does not push on anything. Since the cylinder is insulated, \( Q_A = 0 \), and so \( \Delta U_A = 0 \). Thus the temperature of system A does not change.

The gas in cylinder B does work as it pushes on the piston:
\[ W_B = \int PdV > 0 \]
The heat transfer is still zero, so
\[ \Delta U_B = Q_B - W_B = 0 - W_B < 0 \]
Thus the internal energy, and hence the temperature, of system B decreases.

19.40 The slack bag means that the pressure remains equal to 1 atm. Thus the work done is:
\[ W = P(V_f - V_i) = (1.0 \times 10^5 \text{ Pa}) (1.15 - 1.05) \times 10^3 \text{ m}^3 = 1.0 \times 10^7 \text{ J} \]
The amount of gas in the bag is:
\[ n = \frac{PV}{RT} = \frac{(1.0 \times 10^5 \text{ Pa}) (1.15 \times 10^3 \text{ m}^3)}{(8.3145 \text{ J/mol} \cdot \text{ K}) (270 \text{ K})} = 5.1 \times 10^4 \text{ mol} \]
Since \( U = \frac{3}{2} nRT = \frac{3}{2} PV \), and \( P \) remains constant, then
\[ \Delta U = \frac{3}{2} P \Delta V = \frac{3}{2} W = 1.5 \times 10^7 \text{ J} \]
Then from the 1st law of thermodynamics:
\[ Q = \Delta U + W = 2.5 \times 10^7 \text{ J} \]

19.42 Since the cylinder is insulated, there is no heat transfer. Work is done on the rising block by the system. So:
\[ W = mgh = (10.0 \text{ kg}) (9.8 \text{ m/s}^2) (0.25 \text{ m}) = 24.5 \text{ J} \]
At constant $V$

$$P = \frac{\mathcal{N}RT}{V}$$

Constant $V$ lines are mapped onto straight lines on the $P$-$T$ diagram.

19.44

$$W_{AB} = 5.1 \times 10^4 \text{ J}$$
$$W_{BC} = -2.5 \times 10^4 \text{ J}$$
$$W_{DB} = 0$$
$$W_{DA} = -2.0 \times 10^4 \text{ J}$$

19.45

$$V = 0.60 \text{ m}^3$$
$$T = 2.0 \times 10^2 \text{ K}$$
$$P = 0.050 \text{ atm}$$

a) Use

$$\mathcal{N} = \frac{PV}{RT}$$

$$= \frac{(1.013 \times 10^5 \text{ atm} \times 0.050 \text{ atm})(0.60 \text{ m}^3)}{(8.31 \text{ J/mol-K})(2.0 \times 10^2 \text{ K})}$$

$$= 1.8 \text{ moles}$$

b) now

$$V = 0.50 \text{ m}^3$$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$P_2 = \frac{T_2}{T_1} \times \frac{P_1V_1}{V_2}$$

$$= \frac{210 \text{ K} \times 0.6 \text{ m}^3 \times 0.05 \text{ atm}}{200 \text{ K} \times 0.5 \text{ m}^3}$$

$$= 0.063 \text{ atm}$$

c) No we can’t draw the actual process precisely. It was not isothermal, nor exactly adiabatic. (The product $t’V’$ changes by 5%).

19.46 a) $P = 0.164 \text{ atm}$

b) $W_{(1)} = -1660 \text{ J}; W_{(2)} = -1900 \text{ J}$

c) $T_{(1)} = 200 \text{ K}; T_{(2)} = 256 \text{ K}$

d) $Q_{(1)} = -4160 \text{ J}; Q_{(2)} = -2990 \text{ J}$

19.47

Volume is $V = 0.0160 \text{ m}^3$
Pressure $P = 5.00 \text{ atm}$
Temperature $T = 293 \text{ K}$

Number of moles from

$$\mathcal{N} = \frac{PV}{RT}$$

$$= \frac{(5.00 \text{ atm} \times 0.0160 \text{ m}^3)}{(8.31 \text{ J/mol-K})(293 \text{ K})}$$

$$= 3.33 \text{ moles of Argon}$$

Perform isothermal expansion $\Rightarrow$

$$P_1V_1 = P_2V_2$$

$$V_2 = \frac{P_1V_1}{P_2V_1}$$

$$= \frac{5.00 \text{ atm} \times 0.0160 \text{ m}^3}{3.00 \text{ atm}}$$

$$= 0.027 \text{ m}^3$$

Note, the temperature is still 293° K. The final volume is $V_2 = V_f$

$$V_f = 0.027 \text{ m}^3$$

Now the pressure drops:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$T_2 = \frac{T_1P_2V_2}{P_1V_1}$$

$$= 293 \text{ K} \times \frac{1.00 \text{ atm} \times 0.027 \text{ m}^3}{3.00 \text{ atm}}$$

$$= 97.7 \text{ K}$$

Work was done between the first two points at constant $T$. None is done at constant volume

$$W = \int_1^2 P(V) \, dV$$

$$= \int_{V_1}^{V_2} \frac{\mathcal{N}RT}{V} \, dV$$

$$= \mathcal{N}RT \ln \frac{V_2}{V_1}$$

$$= 425.0 \text{ J}$$