Optical spatial soliton supported by photoisomerization nonlinearity in a polymer with a background beam

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We present detailed theoretical studies of optical spatial solitons (SSs) supported by photoisomerization nonlinearity in a polymer sample. One-dimensional dark and bright SSs and their existence curves are presented. Several combinations of polarizations of the signal and background beams can be used to form the SSs.

1. INTRODUCTION

In the past decade, spatial solitons (SSs) supported by photorefractive effect have attracted much attention. The photorefractive effect is a saturable nonlinearity that can support one- or two-dimensional optical SSs at a very low (microwatts) power level. Interesting discoveries, such as self-trapping of incoherent white light, interaction of the SSs, self-trapping of incoherent light beams, rotating SSs, multiple composite SSs, and discrete solitons have been reported in photorefractive crystals. In a polymer material, several nonlinear effects have been exploited to form SSs and waveguides. The photorefractive effect is a saturable nonlinearity that can support one- or two-dimensional optical SSs at a very low (microwatts) power level. Several combinations of polarizations of the signal and background beams can be used to form the SSs.

2. THEORY AND RESULTS

For the case of unpolarized background radiation, the reaction kinetics of the photoisomerization photochromatic process is described by

\[
\frac{dT_r}{dt} = -q_{T_r} \sigma_{T_r} I_s \cos^2 \theta T' + q_{C_b} \sigma_{C_b} I_s (T_0 - T') - \frac{1}{3} q_{T_b} \sigma_{T_b} I_b T' + q_{C_b} \sigma_{C_b} I_b (T_0 - T') + K(T_0 - T')
\]

for the linearly polarized signal beam and

\[
\frac{dT_0}{dt} = -q_{T_0} \sigma_{T_0} I_s \sin^2 \theta T' / 2 + q_{C_b} \sigma_{C_b} I_s (T_0 - T')
\]

\[
- \frac{1}{3} q_{T_b} \sigma_{T_b} I_b T' + q_{C_b} \sigma_{C_b} I_b (T_0 - T') + K(T_0 - T')
\]

for the circularly polarized signal beam, where \(T_r(T_c)\) and \(T_0\) represent the concentrations of molecules in the trans form under linearly (circularly) polarized illumination and in darkness, respectively; \(\theta\) is the angle between the molecular orientation and the direction of electric field of the signal beam; \(\theta'\) is the angle between the molecular orientation and the direction of the wave vector; \(q_{T_r}\) and \(q_{C_b}\) are the quantum yields of the signal beam for trans-to-cis and cis-to-trans transitions, respectively; \(\sigma_{T_r}\) and...
\( \sigma_{C_b} \) are the absorption cross sections of the signal beam in the trans-to-cis and cis-to-trans transitions, respectively; \( K \) is the thermal relaxation rate of the cis-to-trans transition; and \( I_s \) is the intensity of the light beam. The steady-state concentration is given by

\[
T = T_0 \frac{q_{C_b} \sigma_{C_b} I_s + q_{C_b} \sigma_{C_b} I_b + K}{q_{T_b} \sigma_{T_b} I_s + q_{T_b} \sigma_{T_b} I_b + K}
\]

for a linearly polarized signal beam and

\[
T' = T_0 \frac{q_{C_b} \sigma_{C_b} I_s + q_{C_b} \sigma_{C_b} I_b + K}{q_{T_b} \sigma_{T_b} I_s + q_{T_b} \sigma_{T_b} I_b + K}
\]

for a linearly polarized signal beam. If the signal beam is absent, the steady-state distribution \( T' \) is then given by

\[
T = T_0 \frac{q_{C_b} \sigma_{C_b} I_s + K}{q_{T_b} \sigma_{T_b} I_s + 3q_{C_b} \sigma_{C_b} I_b + K}.
\]

Let \( \alpha = q_{C_b} \sigma_{C_b} / q_{T_b} \sigma_{T_b}, \beta = q_{T_b} \sigma_{T_b} / q_{T_b} \sigma_{T_b}, \gamma = (q_{C_b} \sigma_{C_b} + K / I_b) / (q_{T_b} \sigma_{T_b}), I = I_s / I_b \), then we get

\[
T = T_0 \frac{aI + \gamma}{\beta I \cos^2 \theta + aI + 3/3 + \gamma},
\]

\[
T = T_0 \frac{aI + \gamma}{\beta I \sin^2 \theta + aI + 3/3 + \gamma}.
\]

The refractive index change induced by the signal beam is then

\[
\Delta(n^2) = S \int (T - T') \cos^2 \theta \, d\Omega,
\]

where \( S \) is a constant for a given material and a fixed signal beam wavelength. We get

\[
\Delta(n^2) = 4\pi ST_0 \left[ \frac{aI + \gamma}{\beta I} \left( 1 - \sqrt{\frac{1/3 + \gamma + aI}{\beta I}} \right) \right. \\
\left. \tan^{-1} \left( \sqrt{\frac{\beta I}{1/3 + \gamma + aI}} - \frac{\gamma}{3(\gamma + 1/3)} \right) \right]
\]

for a linearly polarized signal beam and

\[
\Delta(n^2)_b = 4\pi ST_0 \left[ \frac{aI + \gamma}{\beta I} \left( 1 + \frac{aI + 3/3 + \gamma}{\beta I} \right) \ln \left( \frac{2(aI + 3/3 + \gamma)}{\beta I} + 1 \right) \right. \\
\left. - \frac{\gamma}{3(\gamma + 1/3)} \right]
\]

for a circularly polarized signal beam.

For a linearly polarized background and signal beams, the polarization direction of the signal beam can be perpendicular to or parallel to that of the background beam. The steady-state concentration \( T \), according the geometric relation shown in Fig. 1, is given by

\[
T\parallel = T_0 \frac{q_{C_b} \sigma_{C_b} I_s + q_{C_b} \sigma_{C_b} I_b + K}{q_{T_b} \sigma_{T_b} I_s \cos^2 \theta + q_{C_b} \sigma_{C_b} I_s + q_{T_b} \sigma_{T_b} I_b \cos^2 \theta + q_{C_b} \sigma_{C_b} I_b + K},
\]

\[
T\perp = T_0 \frac{q_{C_b} \sigma_{C_b} I_s + q_{C_b} \sigma_{C_b} I_b + K}{q_{T_b} \sigma_{T_b} I_s \cos^2 \theta + q_{C_b} \sigma_{C_b} I_s + q_{T_b} \sigma_{T_b} I_b \cos^2 \theta + q_{C_b} \sigma_{C_b} I_b + K},
\]

where \( T\parallel \) and \( T\perp \) correspond to the cases in which the polarization directions of the signal beam are parallel to and perpendicular to that of the background beam, respectively, and \( \cos^2 \theta = \cos^2 \varphi \sin^2 \Psi \) (Fig. 1). If there is no sig-
According to Eq. (8), the refractive perturbation is given by

\[ \Delta(n^2) = 4\pi S T_0 \left( \frac{2I_0}{n_0^2} + \frac{2I_0}{b_0^2} \right) \]

\[ = 4\pi S T_0 \left( \frac{2I_0}{n_0^2} + \frac{2I_0}{b_0^2} \right) \]

\[ \times \ln \left( \frac{2I_0}{n_0^2} \right) \]

\[ - \gamma \left( 2 + \frac{2\gamma}{\sqrt{2\gamma + 1}} \ln \left( \sqrt{2\gamma + 1} \right) \right) \].

(14)

For a circularly polarized background beam, according to the discussion below, the signal beam should be circularly polarized. The steady-state concentration \( T \) is then given by

\[ T_{ss} = T_0 \frac{q_{cs} \sigma_c I_b + q_{cb} \sigma_b I_b + K}{q_{cs} \sigma_c I_b + q_{cb} \sigma_b I_b + K} \]

where \( \theta' \) is the angle between the molecular orientation and the direction of the wave vector. If the signal beam is absent, the steady-state distribution \( T' \) is then

\[ T' = T_0 \frac{q_{cs} \sigma_c I_b + K}{q_{cs} \sigma_c I_b + q_{cb} \sigma_b I_b \sin^2 \theta'/2 + K} \]

(17)

The refractive perturbation, according to Eq. (8), is given by

\[ \Delta(n^2)_{ss} = 4\pi S T_0 \left( \frac{2I_0}{n_0^2} + \frac{2I_0}{b_0^2} \right) \]

\[ \times \ln \left( \frac{2I_0}{n_0^2} \right) \]

\[ - \gamma \left( 2 + \frac{2\gamma}{\sqrt{2\gamma + 1}} \ln \left( \sqrt{2\gamma + 1} \right) \right) \].

(18)

The wavelengths of the signal and background beams can vary. In this paper, we only discuss two limiting cases, which are easy to realize in experiments. The first case is that the signal and background beams have the same wavelength and are mutually incoherent, and the second case is that the wavelength of the signal beam is in the long-wavelength range, whereas that of the background beam is within the absorption range of the isomerization (for azo-doped material, typically in the blue–green region).

For the first case, we put \( \beta = 1 \) and \( \alpha = \gamma \) (according to Ref. 30, \( K \) can be neglected), and get \( \Delta(n^2)_{ss} = 0 \), \( \Delta(n^2)_{ss} = 0 \), and thus no SS exists. Figure 2 shows \( \Delta(n^2)_{ss} \), \( \Delta(n^2)_{ss} \), and \( \Delta(n^2)_{ss} \) as functions of \( I \) for \( \alpha = 3 \). We see that \( \Delta(n^2)_{ss} \), \( \Delta(n^2)_{ss} \), and \( \Delta(n^2)_{ss} \) are negative and therefore can form
dark SSs. Our calculation shows that when \( \gamma \) is increased, the saturable values of \( \Delta n_{t}, \Delta n_{c}, \Delta n_{l}, \Delta n_{c} \), and \( \Delta n_{cc} \) decrease.

Propagation of the linearly polarized signal beam satisfies the Schrödinger equation

\[
\nabla^2 A + k_0^2 \Delta(n^2) A + 2ik \frac{\partial A}{\partial z} = 0,
\]

where \( A \) is the slowly varying amplitude of the electric field of light. In general, the optical electrical field of the circularly polarized signal beam can be resolved into two components, each satisfying the scalar Schrödinger equation. However, Ref. 33 shows that the slowly varying amplitudes of the two components satisfy the same Schrödinger equation, namely, Eq. (19), and can be assumed to be equal. In this case, \( A \) represents the slowly varying amplitude of the component of the circularly polarized signal beam.

Let \( \Gamma \) be the propagation constant of the SS for the one-dimensional dark SS and let \( A = u(x) \sqrt{J_0} \exp(iTz) \) for the linearly polarized beam and \( A = u(x) \sqrt{J_0}/2 \exp(iTz) \) for the circularly polarized beam, so that \( u^2 = 1 \). We get the SS solutions in dimensionless form, as shown in Fig. 3(a) for \( u(\infty) = 1 \). The existence curves of SSs are shown in Fig. 3(b). We see that the existent curves are similar to those of photorefractive SSs, in which the FWHM of the soliton gets a minimum for a certain value of \( u(\infty) \).

For the second case, we assume that \( \alpha = \beta = 0.1 \), \( \Delta n_{t}, \Delta n_{c}, \Delta n_{l}, \Delta n_{c} \), and \( \Delta n_{cc} \) as functions of \( I \) are shown in Fig. 4(a)–4(d) for \( \gamma = 0.1, 0.7, 1, 2, \) and \( 4 \), respectively. Note that the saturable values of \( \Delta n_{t}, \Delta n_{c}, \Delta n_{l}, \Delta n_{c} \), and \( \Delta n_{cc} \) decrease, from positive to negative, when \( \gamma \) decreases.

![Figure 3](image-url)  
Fig. 3. (a) Soliton solutions, where \( \xi = x/x_0 = (4k_0^2 \pi T)_{1/2} \), (b) the existence curves for \( \beta = 1 \) and \( \alpha = \gamma = 3 \). \( \perp \) corresponds to the case of linearly polarized signal and background beams with polarizations perpendicular to each other; \( l \) and \( c \) correspond, respectively, to linearly and circularly polarized signal beams with an unpolarized background beam.

![Figure 4](image-url)  
Fig. 4. Refractive index perturbation versus intensity of signal beam for \( \alpha = \beta = 0.1 \) and (a) \( \gamma = 0.1 \), (b) \( \gamma = 0.7 \), (c) \( \gamma = 1 \) [for this case \( \Delta(n^2) = \Delta(n^2)_{0} = 0 \)], (d) \( \gamma = 2 \), and (e) \( \gamma = 4 \). \( l \), \( c \), \( c \), \( l \), and \( \perp \) represent \( \Delta(n^2), \Delta(n^2)_{l}, \Delta(n^2)_{c}, \Delta(n^2)_{l}, \) and \( \Delta(n^2)_{c} \), respectively.
3. DISCUSSION

From Section 2, we see that the combinations of polarization signals and background beams considered can form steady-state SSs. In the case shown in Fig. 5(a), when the polarization of the signal beam or background beam is changed, the SS can be switched from bright to dark or vice versa. For example, for the combination of linearly polarized signal and background beams, when the polarization direction of the background is changed from parallel to perpendicular to that of the signal beam, the SS is switched from bright to dark for $\alpha = \beta = 0.1$ and $\gamma = 0.7$.

On the other hand, not all combinations of the polarizations of signal and background beams can form steady-state SSs. For example, for the combination of a circularly polarized signal beam and a linearly polarized background beam, the signal beam should be decomposed into two perpendicular linearly polarized components in the cross section. Owing to the optical induced anisotropic refractive index perturbation, the two linearly polarized components of the signal beam do not satisfy the same Schrödinger equation as treated in Ref. 33 (say spontaneous circularly polarized SS). Then these two components will propagate differently in the medium and thus should be treated as a mutually interacting vector SS pair, not a circularly polarized SS.

In this paper, “unpolarized light” means uniform radiations from all directions, so that the effective absorption coefficient$^{12}$ of the trans isomerization is an average of $\sigma_T \cos^2 \theta$ over all solid angles, which gives $\sigma_T/3$, where $\sigma_T$ is the only nonvanishing component of the diagonal absorption cross-section tensor.

The values of the parameters $\alpha$, $\beta$, and $\gamma$ are determined by the material as well as the wavelengths of the signal and background beams. We choose a disperse red one (DR1)-doped PMMA matrix as an example. The values of $q_{TS}$ and $q_{CS}$ (the quantum yields of the signal beam for trans-to-cis and cis-to-trans transitions, respectively) vary slightly, and $\sigma_C$ (the absorption cross sections of a light beam in the cis-to-trans transitions) have large variations in the visible range.$^{29}$ If the proper wavelengths of the signal and background beams are chosen, we can get the desired values of the parameters. For the case that the wavelengths of the signal and background beams are the same, say 514.5 nm, the value of $\alpha$ is found to be approximately 3. For the case in which the wavelength of the signal beam is in the long-wavelength range, while that of the background beam is in the absorption range of the isomerization, for example, the wavelength of the signal beam is 633 nm and that of the background beam is between 400 and 560 nm, the values of $\alpha$ and $\beta$ are approximately 0.1, and the value of $\gamma$ is between 0.5 and 7. So the parameters chosen in our calculations correspond to realistic parameter values of real materials that can be tested experimentally.

Since absorption of the light beam in the material is significant, we briefly discuss the evolution of the soliton in the material with loss. When a light beam propagates in the material, the normalized evolution equation with loss in the one-dimensional case is

$$\frac{\partial u}{\partial \eta} = \left\{ i \left[ \frac{\partial^2}{\partial \xi^2} + \frac{\Delta(n^2)}{4\pi ST_0} \right] - \Phi \right\} u, \tag{20}$$

where $\xi = \eta/x_0$, $x_0 = (4k_0^2\pi ST_0)^{-1/2}$, $\eta = z/z_0$, $z_0 = k/(2k_0^2\pi ST_0)$, $\Phi = \Omega k/(4k_0^2\pi ST_0)$, and $\Omega$ is the linear absorption coefficient. As an example, consider the case of the linearly polarized signal beam and the unpolarized background beam. Let $\alpha = \beta = 0.1$, and $\gamma = 0.3$, the soliton solution for low intensity ($I = 1$) and high intensity ($I = 16$) are shown in Fig. 6 (solid curve) together with the existence curve of the soliton. Let $ST_0 = 10^{-2}$ (corresponding refractive index change of the order of $10^{-3}$), and

![Fig. 5. Soliton solutions for (a) $\gamma = 0.7$, different combinations of the polarization of signal and background beams, (b) polarizations of signal and background beams perpendicular, $u_+ = 1$ (or $u_0 = 1$), $\alpha = \beta = 0.1$, $\gamma = 0.7$ and 2 [Fig. 5(b)].](image-url)
$\Omega = 70 \text{ m}^{-1}$ (meaning that when a light beam that has passed through a 10 mm sample loses half of its energy). The evolutions of the solitons with low and high intensities are shown in Fig. 7, and their output intensity profiles are shown in Fig. 6 (dashed curve). For the soliton with low intensity, its FWHM increases after propagating a 20 mm distance, whereas the FWHM of the soliton with high intensity is almost unchanged. This can be explained by the existence curve in Fig. 6 as follows. The absorption causes the decrease of the intensity of the solitons, thus the FWHM of the soliton increases rapidly for the case of low intensity, while the FWHM decreases slowly or even remains unchanged for the high-intensity case, which is similar to the case of the photorefractive soliton.  

4. CONCLUSION
We present theoretical results that several combinations of the polarizations of signal and background beams can form steady-state spatial solitons supported by the isomerization nonlinearity in a polymer. In some cases, when the polarization of the signal or background beams is changed, the bright or the dark solitons can be switched. This switching can also be achieved by varying the wavelength of the background beam.

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