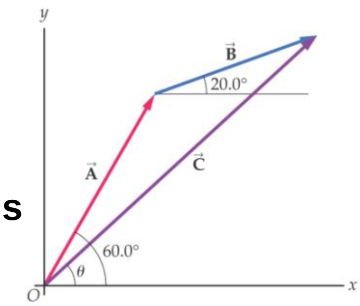
Chapter 3

Vectors in Physics

Is 1+1 always =2?

Not true for vectors.
Direction matters.
Vectors in opposite directions can partially cancel.



Position vectors, displacement, velocity, momentum, and forces are all vectors.
When you add vectors, direction, (angles and negative signs) matters!!!

Units of Chapter 3

- Scalars Versus Vectors
- The Components of a Vector
- Adding and Subtracting Vectors
- Unit Vectors
- Position, Displacement, Velocity, and Acceleration Vectors
- Relative Motion

3-1 Scalars Versus Vectors

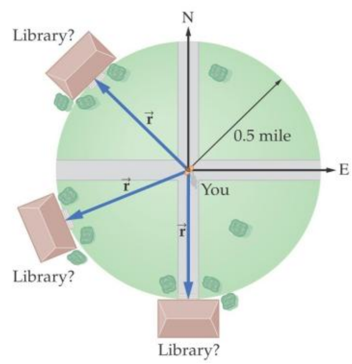
Scalar: number with units. (scalars can be +,-or 0)
Scalars doesn't include direction.

Vector: quantity with magnitude and direction.

How to get to the library: need to know how far and which way

Vector's magnitude are scalars.

Magnitude of Vector's components in x and y directions are scalars.

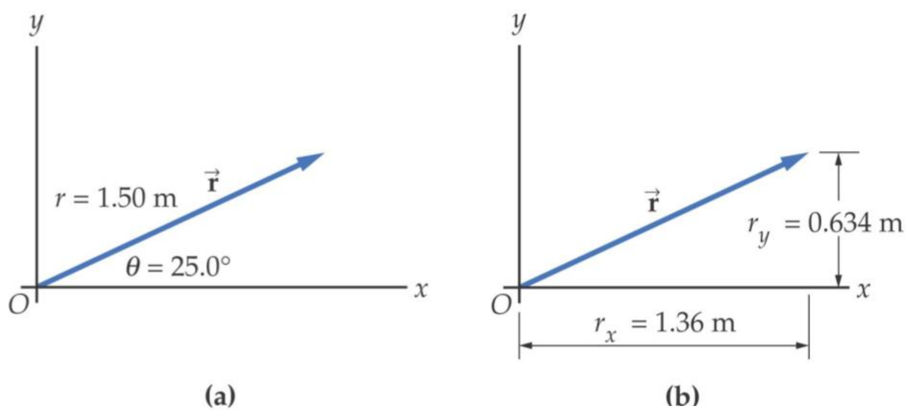


3-2 The Components of a Vector

Even though you know how far and in which direction the library is, you may not be able to walk there in a straight line:

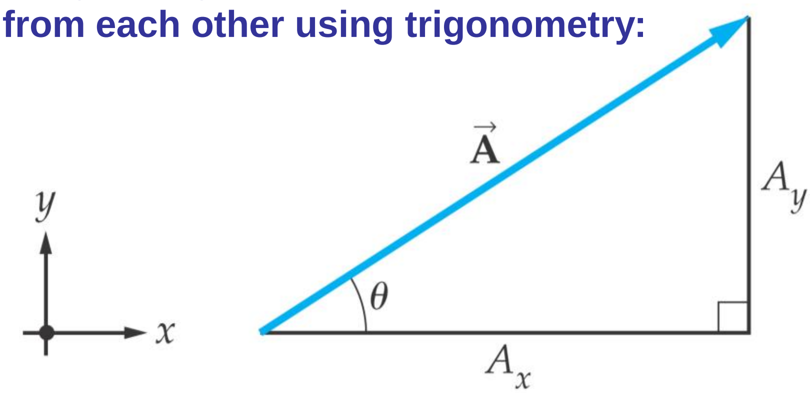
3-2 The Components of a Vector

Can resolve vector into perpendicular components using a two-dimensional coordinate system:



3-2 The Components of a Vector

Length, angle, and components can be calculated from each other using trigonometry:



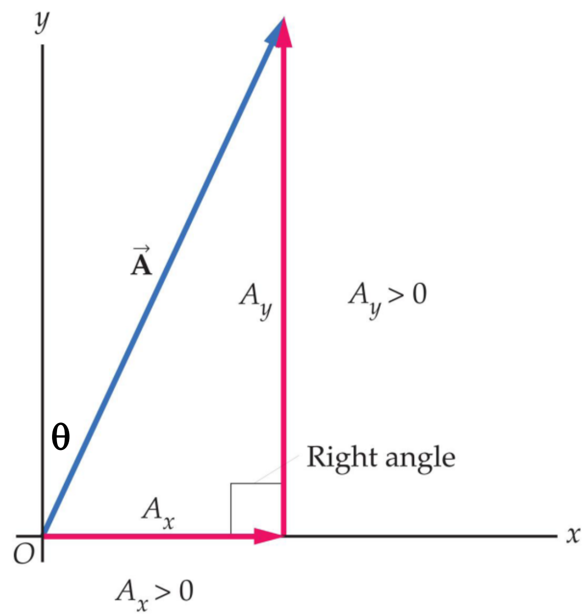
$A_x = A\cos\theta$ $A_y = A\sin\theta$ $A_y / A_x = \tan\theta$

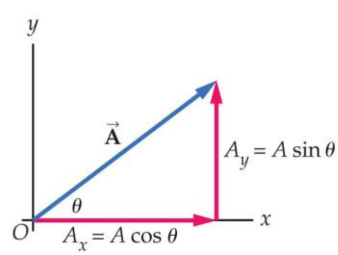
$\theta = \cos^{-1}(A_x / A)$ $\theta = \tan^{-1}(A_y / A_x)$

To decompose a vector, remember to start at the vector's tip and draw lines PERPENDICULAR to x and y axes. Then use trig to solve them.

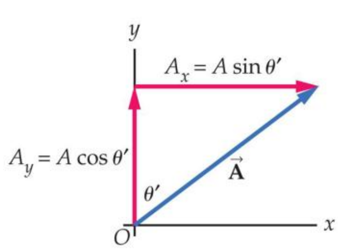
$A_x = A \sin\theta$
$A_y = A \cos\theta$

$A = (A_x^2 + A_y^2)^{1/2}$





(a)



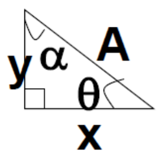
(b)

Never memorize Ax= A cos…

Always do trig Step by step.

Right triangle Review yourself

Memorize them



A sinθ= y A cosα = y

A cosθ= x A sinθ = x

$A^2 = x^2 + y^2$ sinθ = cos α

α + θ= 90° cosθ = sinα

tanθ= y/x

tanθ=ctanα

tanα= x/y

tanα =ctanθ

$1 = \sin\theta^{\,2} + \cos\theta^2$

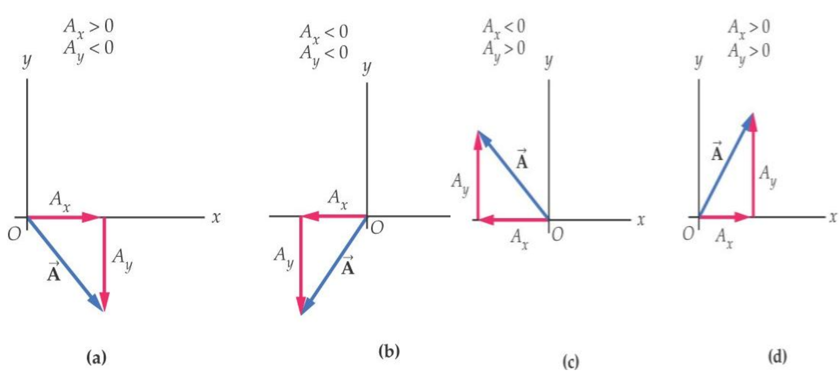
sin(0)=0 sin(90)=1 sin(45)=cos(45)

cos(0)=1 cos(90)=0 cos(180)=-1

tan(0)=1 tan(45)=1=ctan(45)

3-2 The Components of a Vector

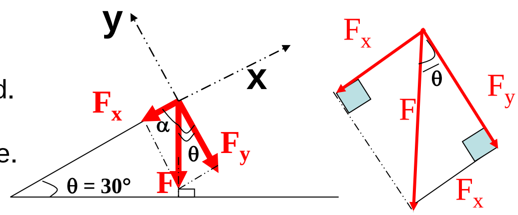
Signs of vector components:



$A = (A_x^2 + A_y^2)^{1/2}$

De-component Force F along x and y directions:

F is in Newton downward. Angle θ=30° between incline and ground.
X direction along incline
Y direction perpendicular to incline.



$$\vec{F} = F_x \hat{x} + F_y \hat{y}$$

Steps:

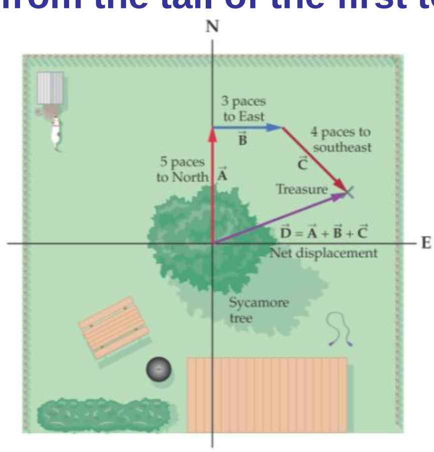
1. From the tip of F's arrow draw a line perpendicular to the x axis (⊥ to the incline).

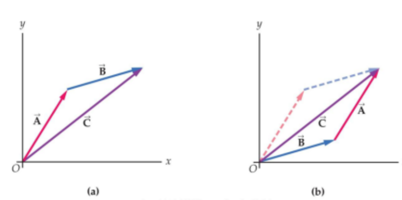
The intersection point tells you the size of F_x.(magnitude of x component).

1. From the tip of F's Arrow draw a line perpendicular to y axis. The intersection point tells you the size of F_y. (magnitude of y component)
2. Realize that the angle between downward F and negative y axis is equal to θ. Because α=90°- θ. Both θ on the graph + α=90°.
3. $|F_x|$=Fsinθ: $|F_y|$=Fcosθ. Fx=-Fsinθ=-10*sin30°=-5.0(Newton)
   F_y=-Fcosθ. Fy=-Fcosθ=-10*cos30°=-8.7(Newton)

3-3 Adding and Subtracting Vectors

Adding vectors graphically: Place the tail of the second at the head of the first. The sum points from the tail of the first to the head of the last.





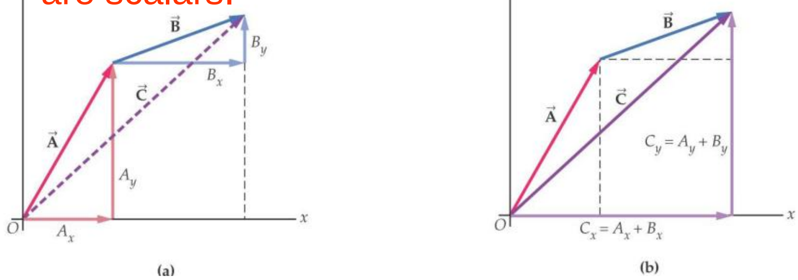
3-3 Adding Vectors

Adding Vectors Using Components: Find the components of each vector to be added. Add the *x*-and *y*-components separately. Find the resultant vector.

$\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$

$C_x = A_x + B_x$;
$C_y = A_y + B_y$

We can add or subtract the vector components directly as numbers, because the components are scalars.



3-3 Subtracting Vectors

Subtracting Vectors: The negative of a vector ($-\vec{\mathbf{B}}$) is a vector of the same magnitude pointing in the opposite direction of vector $\vec{\mathbf{B}}$.

$\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$

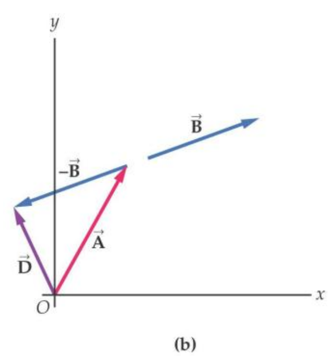
How to find $\vec{\mathbf{D}}$?

1, $\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$ is the same as $\vec{\mathbf{A}} + (-\vec{\mathbf{B}})$

2, Using components

$D_x = A_x - B_x$

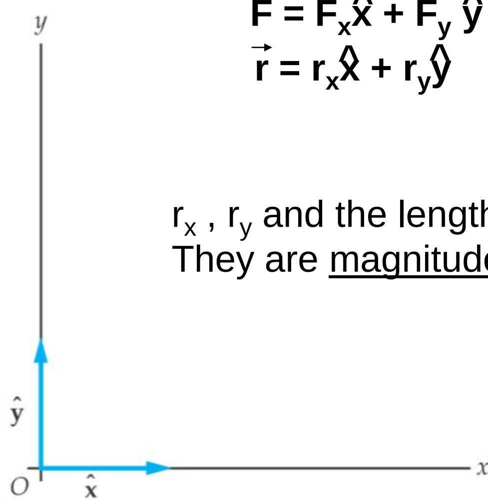
$D_y = A_y - B_y$



3-4 Unit Vectors

Unit vectors are dimensionless vectors of unit length.

$$\vec{F} = F_x\hat{x} + F_y\hat{y}$$
$$\vec{r} = r_x\hat{x} + r_y\hat{y}$$

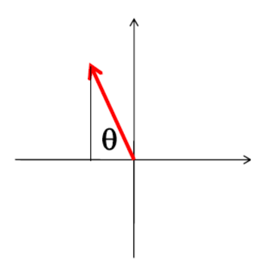


r_x , r_y and the length of r are scalars.
They are magnitude in x, y or r directions.

Find vector B with component B_x= -1 m, B_y = 2m
Answer: The vector size: $B = \sqrt{(B_x^2 + B_y^2)} = \sqrt{5}$

The vector is on which quarter?

Find the absolute value of the angle between this vector and the –x axis



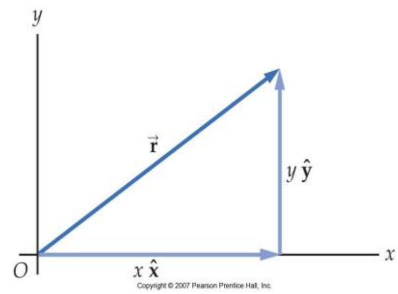
$\cos\theta = (|B_x|/B) = 1/\sqrt{5}$

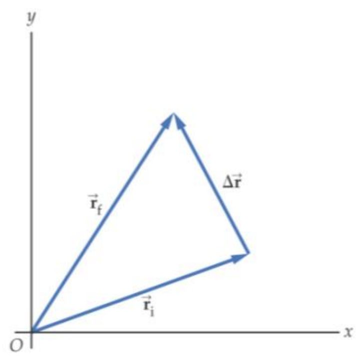
$\sin\theta = (B_y/B) = 2/\sqrt{5}$

$\theta = \tan^{-1}(B_y/|B_x|) = \tan^{-1}(2)$
$= \sin^{-1}(B_y/B) = \sin^{-1}(2/\sqrt{5}) = 63.4$ degree

3-5 Position, Displacement, Velocity, and Acceleration Vectors

Position vector *r* points from the origin to the location in question.





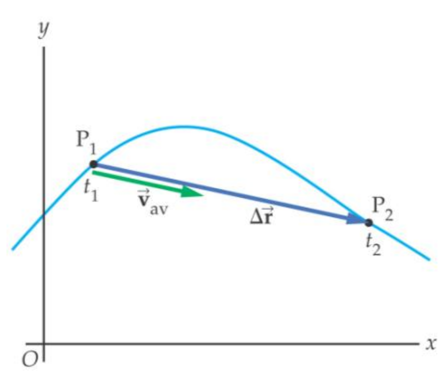
The displacement vector Δ*r* points from the original position to the final position.

3-5 Position, Displacement, Velocity, and Acceleration Vectors

Average velocity vector:

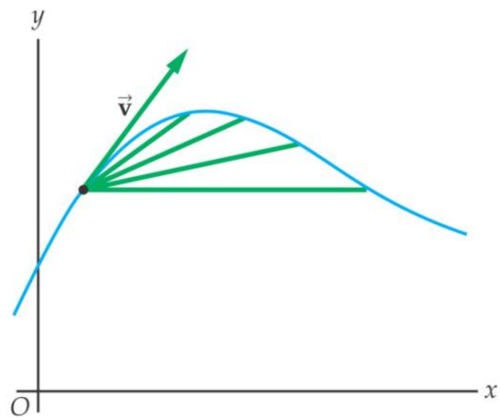
$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} \qquad (3\text{-}3)$$

So v_{av} is in the same direction as Δ*r*.



3-5 Position, Displacement, Velocity, and Acceleration Vectors

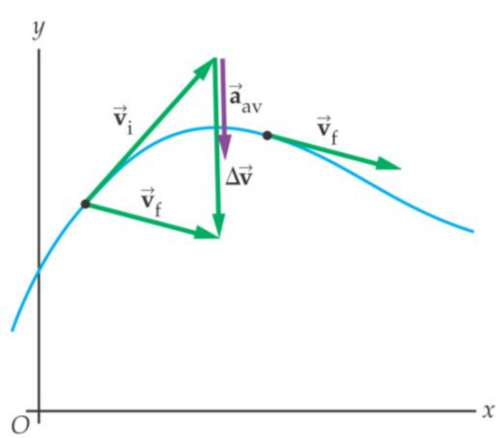
Instantaneous velocity vector is tangent to the path:



3-5 Position, Displacement, Velocity, and Acceleration Vectors

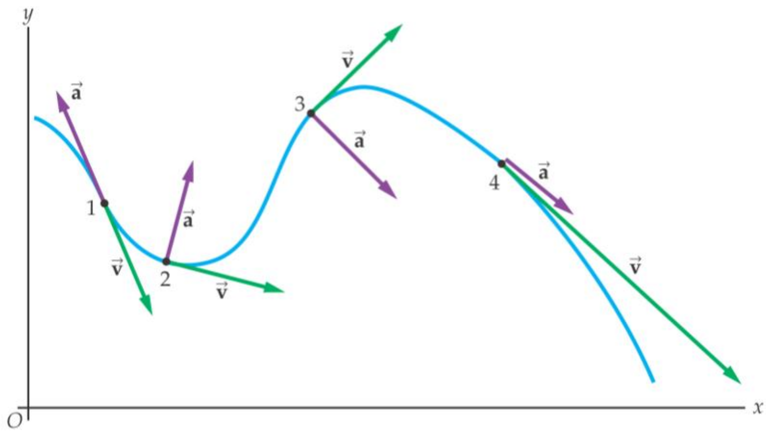
Average acceleration vector is in the direction of the change in velocity:

$$\vec{\mathbf{a}}_{\mathrm{av}} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t}$$



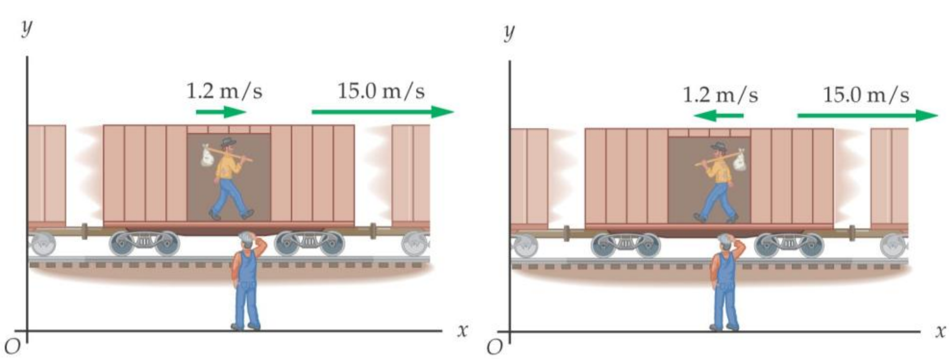
3-5 Position, Displacement, Velocity, and Acceleration Vectors

Velocity vector is always in the direction of motion; acceleration vector can point anywhere:



3-6 Relative Motion

The speed of the passenger with respect to the ground depends on the relative directions of the passenger's and train's speeds:

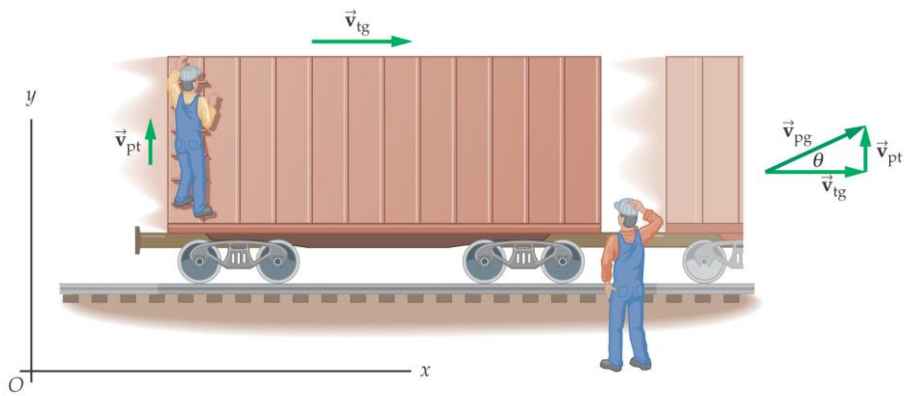


(a) $\vec{\mathbf{v}}_{pg} = \vec{\mathbf{v}}_{pt} + \vec{\mathbf{v}}_{tg}$ (b)

3-6 Relative Motion

$$\vec{\mathbf{v}}_{pg} = \vec{\mathbf{v}}_{pt} + \vec{\mathbf{v}}_{tg}$$

This also works in two dimensions:



Did you ever wonder why the moon seems to follow you which ever direction you go?

So many people in the world, to whom does it really follow?

If you really understand relative motion, vectors, and trigonometry you will solve this puzzle.

(welcome to discuss this puzzle with me in office hours) Hint: Me, tree, moon, distances, angles…

Summary of Chapter 3

- **Scalar: number, with appropriate units**
- **Vector: quantity with magnitude and direction**
- **Vector components:** A_x , A_y
- **Magnitude:** $A = (A_x^2 + A_y^2)^{1/2}$
- **Direction:** $\theta = \tan^{-1} (A_y / A_x)$
- **Graphical vector addition: Place tail of second at head of first; sum points from tail of first to head of last**

Summary of Chapter 3

- **Component method: add components of individual vectors, then find magnitude and direction**
- **Unit vectors are dimensionless and of unit length**
- **Position vector points from origin to location**
- **Displacement vector points from original position to final position**
- **Velocity vector points in direction of motion**
- **Acceleration vector points in direction of change of motion**
- **Relative motion:** $\vec{v}_{13} = \vec{v}_{12} + \vec{v}_{23}$