

Note: numbers used in solution steps can be different from the question part. You can practice the methods in the solution and verify with the numbers and answers given in the question part. Or you should practice the methods in the solution and verify your calculation with numbers in your webassign.

Problem 8-10 are the same questions from previous HWs

Problem 1. A new record for running the stairs of the Empire State Building was set on February 3, 2003. The 86 flights, with a total of 1576 steps, was run in 9 minutes and 33 seconds. Consider a scenario where a man ran up 1600 steps of the Empire State Building in 10 minutes and 59 seconds. If the height gain of each step was 0.20 m, and the man's mass was 80.0 kg, what was his average power output during the climb? Give your answer in both watts and horsepower. (381 W, 0.511 hp)

Solution: the man did work to lift himself up to such a known height within known amount of time.

Power = Work / Time The total time was 10 minutes and 59 seconds =659 s,

What is the total work done by the man?

We know that the work done by the man increased its mechanical energy. $W_{\text{man}} = W_{\text{nc}} = E_f - E_i$

Before he starts and after he stops the velocity was both zero, and KE were both zero.

So the work done by the man is

$$W_{\text{man}} = W_{\text{nc}} = E_f - E_i = mgh_f - mgh_i = mg h_f = 80 \cdot 9.8 \cdot (0.2 \cdot 1600) = 250880 \text{ J}$$

The man spent 659 seconds to do 250880 J of work. This work increased his potential energy by 250880 J. (His height was increased for 320 m)

$$\text{Power} = \text{Work} / \text{Time} = 250880 / 659 \text{ (J/s)} = 250880 / 659 \text{ (Watt)} = 381 \text{ W}$$

$$1 \text{ hp} = 746 \text{ W}$$

$$381 \text{ W} = 381 / 746 \text{ hp} = 0.511 \text{ hp}$$

It looks like the fast or strongest man can be as "powerful" as a half horse. ☺

A forum (under communication) was created within Webassign. I posted student question and my answers there. I wrote there: notice that since it is not constant acceleration, you will not be able to find out the instantaneous velocity at particular moments. You can only find v average. You should not use v average to calculate KE initial or KE final or KE at one point or one particular moment.

In this one, KE at the very initial point and the stopping point are considered to both be zero.

Problem 2, A kayaker paddles with a power output of 47.0 W to maintain a steady speed of 1.50 m/s.

(a) Calculate the resistive force exerted by the water on the kayak. (31.3 N)

(b) If the kayaker increases her power output by a factor of 1.4, and the resistive force due to the water remains the same, by what factor does the kayaker's speed change? (1.4)

Solution: At the beginning you may be surprised to see that the man was paddling hard, constantly do work to the kayak, why is the speed unchanged? Why didn't KE increase? Don't forget that W_{nc} include not only work done by human, but also work done by resistance. There must be resistance force which did negative work, as a result the total $W_{\text{nc}}=0$ and KE didn't change.

The total net force must also be zero, human paddling force canceled with the resistance force.

a). Ok, consider the man, his power is 47.0. With a steady speed and steady force.

$$\text{Power} = \text{Work} / \text{time} = \text{Force} \cdot \text{distance} / \text{time} = \text{Force} \cdot \text{speed}$$

$$\text{Hence, Force} = \text{Power} / \text{speed} = P / v = 47.0 \text{ Watt} / (1.5 \text{ m/s}) = 31.3 \text{ N.}$$

Attention this P is power, not the momentum in chapter 9. Momentum $p = mv$

b). Now, if the resistive force is not changed, the paddling/pushing force is also not change.

$$\text{Speed} = \text{Power} / \text{force}, \text{ When power is 1.4 times larger, your speed is 1.4 times larger.}$$

Attention: here we consider constant speed. Pushing force = resistance force. We are not considering constant accelerating.

Actually when you paddle, in order to output more power, you will have to paddle faster.

Problem 3. A block of mass m and speed v collides with a spring, compressing it a distance Δx . What is the compression of the spring if its force constant is increased by a factor of four? Give your answer in terms of Δx . ($0.5 \Delta x$)

Solution or Explanation

The work done in compressing the spring reduces the Kinetic energy of the object from $\frac{1}{2} m v^2$ to zero.

$$\text{Initial total Energy} = \frac{1}{2} m v^2 = \text{Final total energy} = \frac{1}{2} k (\Delta x)^2$$

, where k is the spring constant of the spring. If the spring constant is increased by a factor of four, the compression distance must be halved to produce the same amount of work or potential energy; that is, $\frac{1}{2} (4k) (\Delta x/2)^2 = \frac{1}{2} k (\Delta x)^2$. Thus, the compression of the spring is $\Delta x/2$.

This question is asking you, when you have a new spring which is 4 times stronger comparing with the old one, in order to store the same amount of potential energy (or in order to do the same amount of work to take away the kinetic energy and stop the same object,) whether the new and stronger spring needs to be compressed more or compressed less than the old spring? (how does the new Δx compare to the old Δx .)

It is always good to first decide the new item will be more or less than the old item. Before plugging numbers. It is easy to realize the new Δx needs to be less than the old Δx , because the new spring is stronger.

The next step is to realize the new Δx is how many times less than the old Δx .

4 times less compression, because the spring is 4 times stronger?

No, No, No, don't forget that there is $(\Delta x)^2$ in the spring potential energy $U = \frac{1}{2} k (\Delta x)^2$

When the new (Δx) is 0.5 times of the old (Δx) , the new $(\Delta x)^2$ square is only $\frac{1}{4}$ of the old $(\Delta x)^2$

With the new spring constant to be 4 times stronger, the new compression distance (Δx) only needs to be half of the old compression amount, to store the same amount of Potential energy.

Also, because $U = \frac{1}{2} k (\Delta x)^2$, $(\Delta x) = \sqrt{2U/k}$, if U is the same, and k becomes 4 times,

(Δx) is proportional to $\sqrt{1/k}$, the new (Δx) is then $\sqrt{1/4} = 1/2$ times of the old (Δx) .

These kind of questions are very important. Practice algebra, and get them right.

If you can not do it right, the last thing you can do is to make up some numbers for U , k , new k , (Δx) etc, and find out new (Δx) , then compare with old (Δx) .

Problem 4, A system of particles is known to have zero momentum. Does it follow that the kinetic energy of the system is also zero?

No!!!

Consider, for example, a system of two particles. The total momentum of this system will be zero if the particles move in opposite directions with equal momentum. The kinetic energy of each particle is positive, however, and hence the total kinetic energy is also positive. Keep in mind that momentum is vector and opposite directions cancel when they add together. KE is scalar, no matter which direction the motion is, KE are always not negative.

Problem 5, Object A has a mass m , object B has a mass $4m$, and object C has a mass $m/4$. Rank these objects in order of increasing momentum, given that they all have the same kinetic energy. Indicate ties where appropriate. (Use only the symbols $<$ and $=$, for example $A < B=C$.)

($C < A < B$)

See the book and lecture notes. Recall that kinetic energy is related to momentum as follows: $K = p^2/2m$. It

follows, then, that $p = \sqrt{2mK}$. Given that the kinetic energy of each of these objects is the same, their

momentum varies as the square root of the mass. Applying these considerations, we have the following ranking:

$C < A < B$. Again, if you don't understand this math, make up some numbers and compare them.

Problem 6, Object A has a mass m , object B has a mass $2m$, and object C has a mass $m/2$. Rank these objects in order of increasing kinetic energy, given that they all have the same momentum. Indicate ties where appropriate. (Use only the symbols $<$ or $=$, for example $A < B=C$.) Answer: ($B < A < C$ -or- $C > A > B$)

Recall that kinetic energy can be written as $p^2/2m$. Since these objects have the same momentum, it follows that their kinetic energies are inversely proportional to their masses. Hence, we have $B < A < C$

What does that mean: Recall my explanation in Tue. lecture:

When p are the same, larger m object has less velocity. While $K=1/2 m v^2$,

m is not that important to KE comparing to v , because v is squared in K , v is more important.

When p are the same, larger m object has less velocity, hence Less K .

Mass B is twice of mass A, when they have the same p , Velocity B is only $1/2$ of velocity A.

Velocity square for object B is only $1/4$ of Velocity square for object A.

K for object B end up only half of K for object A. (now you know, when larger mass shares the same amount of momentum, that means the KE is less than the small mass object with the same p .)

If you are not very good at this kind of math, it is always very helpful to make up some numbers

$m=1, 2$ or 4 kg, $p=4$ kg m/s, and K will be how many J for each of them?

(velocity is how many m/s for each of them?)

Problem 7, A net force of 200 N acts on a 100 kg boulder, and a force of the same magnitude acts on a 100 g pebble. Is the change of the boulder's momentum in one second greater than, less than, or equal to the change of the pebble's momentum in the same time period?

Equal to.

The boulder and the pebble have the same rate of momentum change, since the same force acts on both objects. Force, in fact, is the rate of change of momentum. The larger the force is, the quicker the momentum will change.

Problem 8, A net force of 200 N acts on a 100 kg boulder, and a force of the same magnitude acts on a 100 g pebble. Is the change in the boulder's speed in one second greater than, less than, or equal to the change in speed of the pebble in the same time period?

Less than

The rate of change in momentum is the same for both objects. As a result, the rate of change in velocity for the less massive object (the pebble) must be greater than it is for the more massive object (the boulder).

Alternatively, we know that the acceleration (rate of change of velocity) of an object is proportional to the force acting on it and inversely proportional to its mass. These objects experience the same force, and therefore the less massive object has the greater acceleration.

Problem 9, Find the magnitude of the impulse delivered to a soccer ball when a player kicks it with a force of 1395 N.

Assume that the player's foot is in contact with the ball for 5.90×10^{-3} s. 8.23 kg·m/s

Solution: The force of the kick acting over a period of time imparts an impulse to the soccer ball.

Impulse $I=F \cdot \Delta t = 1350 \cdot 5.90 \cdot 10^{-3} = 8.23 \text{ N}\cdot\text{s} = 8.23 \text{ kg}\cdot\text{m/s}$

(Don't forget unit of force $1 \text{ N} = 1 \text{ kg} \cdot 1 \text{ m/s}^2$) It is convenient, when you use SI unit in your equations, you always end up get SI unit for that item.

Problem 10,11 are from previous HW and the solutions were out for a while. These are important questions and you should be able to do them without problem, after seeing the solution. Any question you didn't answer correctly in your first time deserves a second visit. 😊

Problem 10,

A 40 kg block slides with an initial speed of 15 m/s up a ramp inclined at an angle of 45° with the horizontal. The coefficient of kinetic friction between the block and the ramp is 0.5. Use energy conservation to find the distance along

the incline that the block slides before coming to rest. 10.8 m

(Hint: You will need to relate the distance slid to the height attained.)

E_f = E_i + W_{nc} Remember to relate the height raise with the distance slide. H = d * sin θ

Solution: E_f = E_i + W_{nc} Here W_{nc} is not zero, because there is kinetic friction.

Friction is kinetic, f_k = μ_k * N, N = mg * cosθ = mg * cos(45),

f_k = μ_k * mg * cos(45) = 0.5 * 40 * 9.81 * cos(45) = 139 N.

W_{NC} = W_{friction} = f_k * d * (-1) = - f * d

Lowest point U_i = 0, K_i = 1/2 mv_i²

Maximum height, K_f = 0, U_f = mgh

E_f = E_i + W_{nc} So, mgh = 1/2 mv_i² - f * d

Now we need to solve d, We know everything already except for h.

But as the hint told us, we should relate the distance with the height.

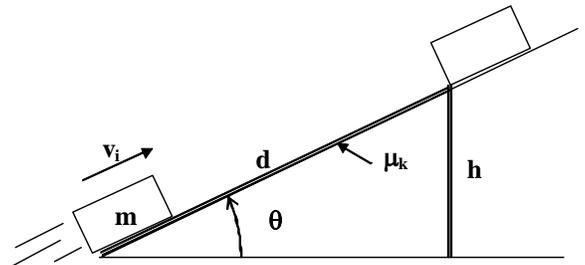
Simply h = d * sin(θ)

So we can solve for d: mg * d * sin(θ) = 1/2 mv_i² - f * d

1/2 mv_i² = f * d + mg * d * sin(θ) = d * (f + mg sin(θ))

d = (1/2 mv_i²) / (f + mg sin(β)) = (1/2 * 40 * 15²) / (139 + 40 * 9.81 * sin(45)) = 10.8 m

Insight: 1/2 mv_i² = f * d + mgh makes sense, right? The total initial kinetic energy was partly converted to mgh and partly taken away by friction. (Notice that the mass cancels out; this is normal, since if you cut the block in half and slid the two pieces up the plane side by side at the same speed, they would go just as far.)



Problem 11, E_f = E_i + W_{nc}

A 1.80 kg block slides on a rough, horizontal surface. The block hits a spring with a speed of 2.00 m/s and compresses it a distance of 11.0 cm before coming to rest. If the coefficient of kinetic friction between the block and the surface is μ_k =

0.560, what is the force constant of the spring? 415 N/m

Picture the Problem: The block slides horizontally on a rough surface, encounters a spring, and compresses it a distance of 11.0 cm before coming to rest.

Strategy: The nonconservative work done by friction changes the mechanical energy of the system. Use equation 8-9 to find ΔE and set it equal to the work done by friction. Solve the resulting expression for the spring constant k.

Solution: 1. Write equation 8-9 to obtain an expression for W_{nc}:

$$W_{nc} = E_f - E_i = (K_f + U_f) - (K_i + U_i) \\ = (0 + \frac{1}{2} kx^2) - (\frac{1}{2} mv_i^2 + 0)$$

2. The nonconservative work is done by friction as the block travels a distance x:

$$W_{nc} = -f_k d = -\mu_k mg x$$

3. Substitute the expression from step 2 into step 1 and solve for k:

$$-\mu_k mg x = \frac{1}{2} kx^2 - \frac{1}{2} mv_i^2 \\ k = \frac{mv_i^2 - 2\mu_k mg x}{x^2} = \frac{m[v_i^2 - 2\mu_k g x]}{x^2} \\ = \frac{(1.80 \text{ kg})[(2.00 \text{ m/s})^2 - 2(0.560)(9.81 \text{ m/s}^2)(0.110 \text{ m})]}{(0.110 \text{ m})^2} \\ k = \boxed{415 \text{ N/m}}$$

Insight: The force exerted by the spring at x = 0.110 m is -45.7 N. Verify for yourself that if the coefficient of static friction with the rough surface is less than μ_s = 2.59, then the spring is strong enough to accelerate the block again in the opposite direction and the block will not remain at rest.