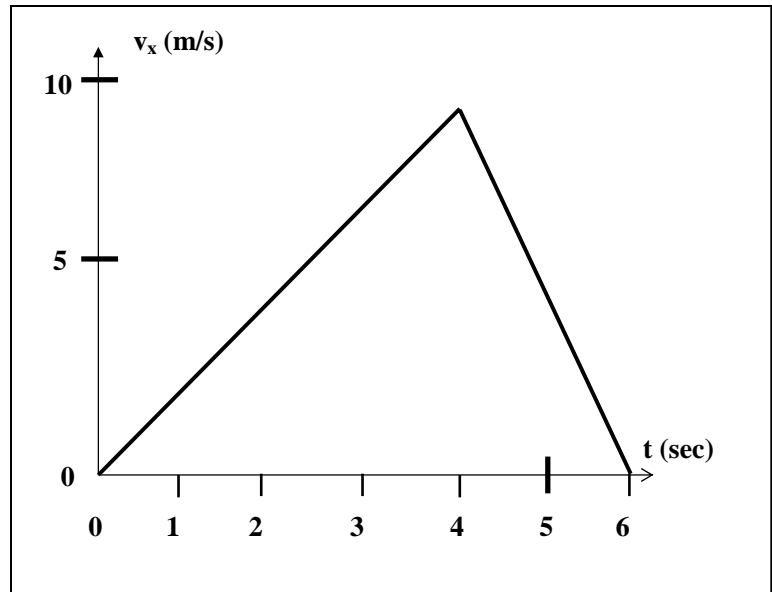


Problem 3 of this assignment - graphing problem. Please note that since I can't actually have you submit a graph in WebAssign, this problem instead asks you to answer questions about what your graph looks like. When asked about the shape of the graph, please use the following choices:



Bob starts off at rest at $x=0$. Then, Bob increases his speed at a constant rate, so that after 4 s, he is travelling 9 m/s in the $+x$ dir. Bob then decreases his speed at a constant rate for 2 s, ending up back at rest.



a.) Draw Bob's v_x vs. t graph, and use it to answer the following:

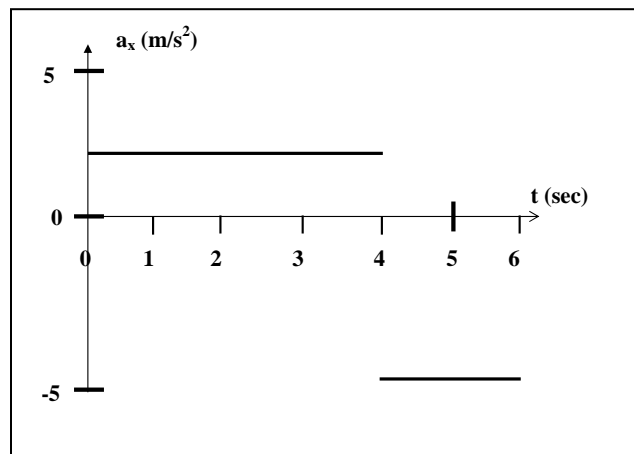
- at $t=0$ s, $v_x = 0$ m/s
- from 0 s to 4 s, this graph is most nearly shaped like: B
- at $t=4$ s, $v_x = 9$ m/s
- from 4 s to 6 s, this graph is most nearly shaped like: C
- at $t=6$ s, $v_x = 0$ m/s

Answers:

- At $t = 0$ Bob is at rest, so the answer is 0 m/s.
- From 0 to 4 seconds the graph goes steadily upwards, like B.
- At 4 seconds the graph peaks out at 9 m/s.
- From 4 to 6 seconds the graph goes steadily down, like
- And at the end, the velocity is down to zero.

b.) Draw Bob's a_x vs. t graph, and use it to answer the following:

- from 0 s to 4 s, this graph is most nearly shaped like:
- and $a_x =$ m/s^2
- from 4 s to 6 s, this graph is most nearly shaped like:
- and $a_x =$ m/s^2



Answers.

- There are two periods of constant

acceleration, first with $a=(9 \text{ m/s})/(4 \text{ s}) = 2.25 \text{ m/s}^2$, then with $a = (-9 \text{ m/s})/(2 \text{ s}) = -4.5 \text{ m/s}^2$. So, the answers are **A**, 2.25 m/s^2 , **A**, and (minus) 4.5 m/s^2 , respectively.

c.) Find Bob's position at $t = 4 \text{ s}$ and 6 s , using graphical methods.

Answers. This means using the fact that the area under the v vs t graph is equal to the distance traveled. This is based on $v = \frac{\Delta x}{\Delta t}$, giving

$$\Delta x = (v)(\Delta t) = (\text{area of a rectangle on the } v \text{ vs } t \text{ graph})$$

So, from 0 to 4 seconds, the v vs t graph is a triangle, with area equal to 1/2 times the base times the altitude:

$$\Delta x(0 \text{ to } 4 \text{ sec}) = \frac{1}{2}(4 \text{ s})(9 \text{ m/s}) = 18 \text{ m}$$

And, from 4 to 6 seconds, there is an additional displacement of

$$\Delta x(4 \text{ to } 6 \text{ sec}) = \frac{1}{2}(2 \text{ s})(9 \text{ m/s}) = 9 \text{ m}$$

d.) Repeat part c, using the constant acceleration eqns. (*This is a good doublecheck that you did part c right.*)

Answers. For $t = 0$ to 4 seconds: initial position = 0 m, initial velocity = 0 m/s, acceleration = $+2.25 \text{ m/s}^2$. Thus

$$\begin{aligned} x(\text{at } t = 4 \text{ sec}) &= x_0 + v_0t + \frac{1}{2}at^2 \\ &= 0 + 0 + \frac{1}{2}(2.25 \text{ m/s}^2)(4 \text{ s})^2 \\ &= 18 \text{ m} \end{aligned}$$

The equations for constant acceleration only apply individually to the two parts of this motion. So, for the second part, we have the time start over at zero, and the initial position is zero, the initial velocity is 9 m/s, and the acceleration is -4.5 m/s^2 . So

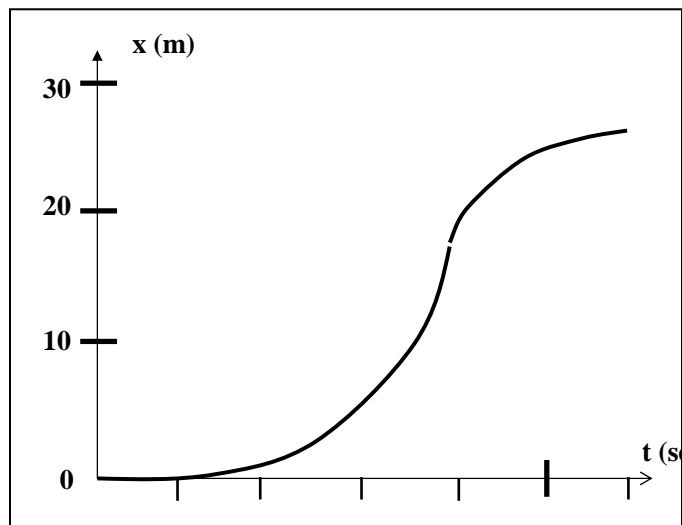
$$\begin{aligned} \Delta x(\text{from } 4 \text{ to } 6 \text{ sec}) &= x_0 + v_0t + \frac{1}{2}at^2 \\ &= 0 + (9 \text{ m/s})(2 \text{ s}) + \frac{1}{2}(-4.5 \text{ m/s}^2)(2 \text{ s})^2 \\ &= 9 \text{ m} \end{aligned}$$

This agrees with the graphical method.

e.) Draw Bob's x vs. t graph.

Use your results from parts c-e to answer the following:

- at $t=0 \text{ s}$, $x =$ m
- from 0 s to 4 s, this graph is most nearly shaped like:
- at $t=4 \text{ s}$, $x =$ m
- from 4 s to 6 s, this graph is most nearly shaped like:
- at $t=6 \text{ s}$, $x =$ m



Answers. The parts of the curve are episodes of constant acceleration, so they have to look like parabolas. Speeding up curves upwards, slowing down curves downwards.

So:

- At $t = 0$, $x = 0$.

- Then curve upwards, like **G**.
- At $t = 4$ s, $x = 18$ m.
- Then curve downwards, like **E**.
- At the end, $x = 27$ m.

Wow! I got 'em all right the first time!

Problem 2. A cannonball is fired out of a cannon at [50] m/s at [45] degrees above horizontal, on level ground.

- How long will the cannonball be in the air?
- What is the maximum height above the ground that it will reach?
- How far will the cannonball travel horizontally?

Solution. We have to make a drawing that shows the components of the initial velocity.

$$\begin{aligned} v_{0x} &= v_0 \cos \theta \\ &= (50 \text{ m/s}) \cos(45^\circ) = 35.36 \text{ m/s} \\ v_{0y} &= v_0 \sin \theta \\ &= (50 \text{ m/s}) \sin(45^\circ) = 35.36 \text{ m/s} \end{aligned}$$

(They happen to be the same because the angle is exactly 45 degrees.)

(a) We will calculate the time to get to the top of the path, using $v_y = v_{0y} - gt$, with $v_y = 0$ and $v_{0y} = 35.36$ m/s. This gives

$$t = \frac{v_{0y}}{g} = \frac{(35.36 \text{ m/s})}{(9.8 \text{ m/s}^2)} = 3.61 \text{ s}$$

The total time in the air is twice this, or 7.22 s.

(b) We will use the equation for y , with $y_0 = 0$ and $v_{0y} = 35.36$ m/s:

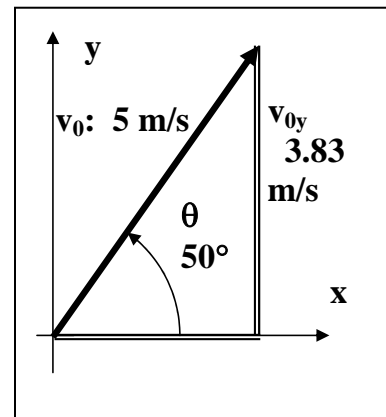
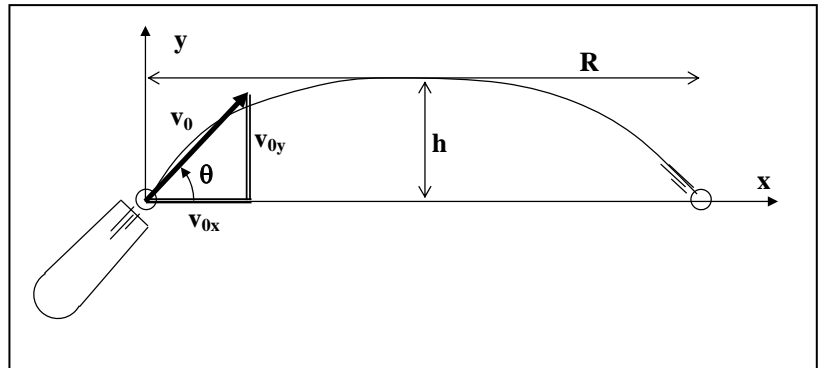
$$\begin{aligned} y &= v_{0y}t - \frac{1}{2}gt^2 \\ &= (35.36 \text{ m/s})(3.61 \text{ s}) - \frac{1}{2}(9.8 \text{ m/s}^2)(3.61 \text{ s})^2 \\ &= 63.79 \text{ m} \end{aligned}$$

(c) The horizontal distance traveled is just the horizontal velocity, equal to v_{0x} , times the total time in the air.

$$x = v_{0x}t = (35.36 \text{ m/s})(7.22 \text{ s}) = 255.3 \text{ m}$$

Problem 3. A ball is thrown off a [37] m high cliff, at [7] m/s, 50 degrees above the horizontal. How long does it take to hit the ground?

Solution. This is a deceptively easy problem. In the usual projectile problem, the first step is to consider the



vertical part of the motion only, independent of the horizontal motion. And the first step of that part is to calculate the time to reach the end of the motion. This have to do for this problem.

First we calculate the vertical component of the initial velocity, $v_{0y} = v_0 \sin 50^\circ = 3.83 \text{ m/s}$

Then we use the equation for the vertical position, with $m, y = 0$, and $v_{0y} = 3.83 \text{ m/s}$.

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2 = 37 + 3.83t - 4.9t^2 = 0$$

or

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2 = 37 + 3.83t - 4.9t^2 = 0$$

$$4.9t^2 - 3.83t - 37 = 0$$

We solve for t using the quadratic equation.

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2 = 37 + 3.83t - 4.9t^2 = 0$$

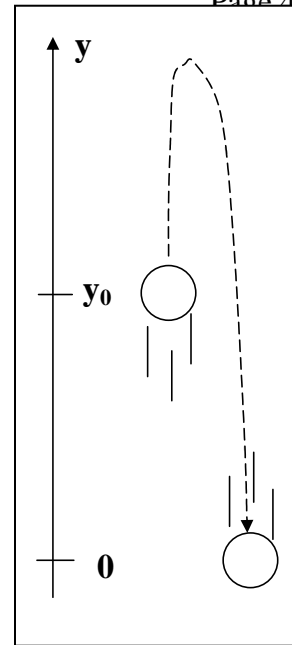
$$4.9t^2 - 3.83t - 37 = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-3.83 \text{ m/s}) \pm \sqrt{(-3.83 \text{ m/s})^2 - 4(4.9 \text{ m/s}^2)(-37 \text{ m})}}{2(4.9 \text{ m/s}^2)}$$

$$= \frac{3.83 \pm 27.20}{9.8} \text{ s}$$

$$= 3.166 \text{ s}$$



is all we
velocity,
 $y_0 = 37$

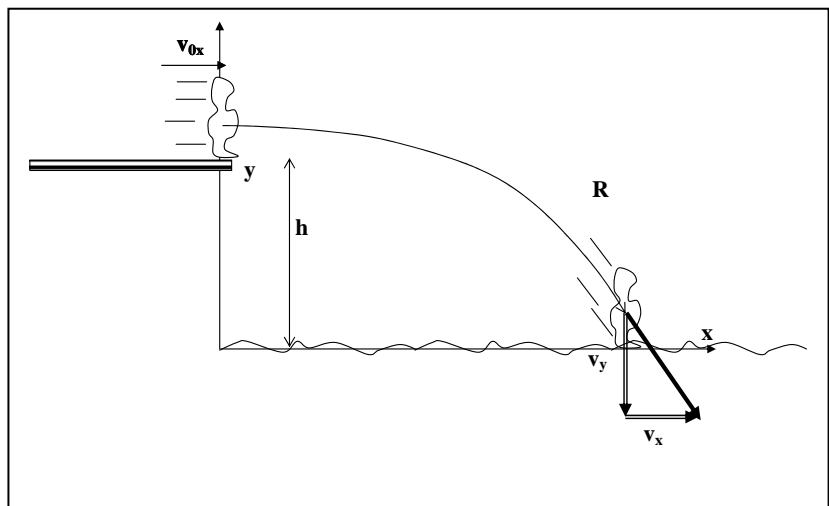
Problem 4. A diver runs horizontally off the end of a [5] m high diving board at [10] m/s.

- (a) How long does it take for the diver to hit the water?
- (b) How far horizontally does the diver end up from the board?
- (c) What is the *size* of the horizontal (x) component of the diver's velocity just before he hits the water?

(d) What is the *size* of the vertical (y) component of the diver's velocity just before he hits the water?

(e) What is the speed of the diver just before he hits the water?

(f) If the diver instead horizontally ran off the end at 12 m/s , which of the answers for a-e would change? (Select all that apply to get credit. Hint: you should not have to re-compute anything to answer this.)



Solutions. (a) This is just the time necessary to fall from rest from the diving board down to the water:

$$\text{distance} = \frac{1}{2}gt^2$$

$$\Rightarrow t = \sqrt{\frac{2d}{g}} = \sqrt{\frac{2(5 \text{ m})}{(9.8 \text{ m/s}^2)}} = 1.010 \text{ s}$$

(b) Now the horizontal travel is just the horizontal velocity (it is constant) times this time:

$$x = v_{0x}t = (10 \text{ m/s})(1.010 \text{ s}) = 10.1 \text{ m}$$

(c) The horizontal component of velocity does not change; it is still 10 m/s.

(d) The vertical component is just equal to g times the time the diver was in the air:
 $v_y = gt = 9.8 \text{ m/s}^2 * 1.01 \text{ s} = 9.9 \text{ m/s}$.

(e) The speed is obtained from the Pythagorean theorem, with horizontal and vertical components just determined:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(10.0 \text{ m/s})_x^2 + (9.9 \text{ m/s})_y^2} = 14.07 \text{ m/s}$$

(f) Which of these would change if the initial horizontal velocity was changed? Well, the vertical motion is independent of the horizontal velocity, so the answers to parts (a) and (d) do not change. The others do.