

Word problems and conceptual problems are very important. We assign do not grade those word problems automatically, but those can be in exams and quizzes. Include those key words highlighted here!

Problem 1. A hand holding a rope moves up and down to create a transverse wave on the rope. The hand completes an oscillation in **2 s**, and the wave travels along the string at **0.5 m/s**. The amplitude of the wave is **0.09 m**.

- a.) Find the frequency at which the crests pass a given point in space. Hz
- b.) Find the distance between two adjacent crests on the wave. m
- c.) There is a blue spot drawn onto the rope with a magic marker. Find the distance this spot travels in one period. m
- d.) If the mass per unit length of the string is $5 \cdot 10^{-4}$ kg/m, what is the tension in the string? N

Solution. The equation for a transverse wave on a string is the "harmonic wave function,"

$$y(x, t) = A \cos\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right).$$

- (a) We are given the period T , from which we can calculate the frequency f :

$$f = \frac{1}{T} = \frac{1}{(2 \text{ s})} = 0.5 \text{ Hz}$$

- (b) This is the wavelength λ , which we can calculate from the frequency and the wave velocity v :

$$\lambda = \frac{v}{f} = \frac{(0.5 \text{ m/s})}{(0.5 \text{ Hz})} = 1.0 \text{ m}$$

- (c) This has to do with the amplitude, given as $A = 0.09 \text{ m}$. During one period, the spot travels up a distance A , then down a distance of $2A$, and then back up a distance A to where it started. (This supposes that the spot started at $y = 0$ initially.) So the distance traveled is

$$\text{distance} = 4A = 4(0.09 \text{ m}) = 0.36 \text{ m}$$

- (d) We are given the mass density $\mu = M/L$, which occurs in the expression for the wave velocity,

$$v = \sqrt{\frac{T}{\mu}}.$$

Solving for the tension gives

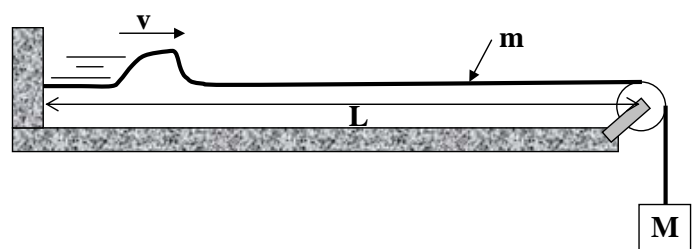
$$T = \mu v^2 = (5 \times 10^{-4} \text{ kg/m})(0.5 \text{ m/s})^2 = 1.25 \times 10^{-4} \text{ N}$$

Problem 2. An astronaut on a small planet wishes to measure the local value of g by timing pulses traveling down a wire which has a large mass suspended from it. Assume that the wire has a mass of **4.00 g** and a length of **1.60 m** and that a **3.00 kg** mass is suspended from it. A pulse requires **35.2 ms** to traverse the length of the wire. Calculate the local g from these data. (You may neglect the mass of the wire when calculating the tension in it.) m/s^2

Solution. The speed v of the pulse down the wire tells us what the tension is, and this tells us the value of g_{local} .

If t is the transit time of the pulse and L the length of the wire,

$$v = \frac{L}{t}$$



The speed is also given by the tension force T , and the mass density μ

$$v = \sqrt{\frac{T}{\mu}},$$

where

$$T = Mg_{\text{local}}$$

is the tension in the wire, maintained by the gravitational force on the hanging mass. And we know the mass density μ , (mass per meter), in terms of the mass and length of the wire:

$$\mu = \frac{m}{L}.$$

So, putting this all together,

$$v = \frac{L}{t} = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Mg_{\text{local}}}{\frac{m}{L}}} = \sqrt{\frac{Mg_{\text{local}}L}{m}}$$

Squaring and solving for g_{local} gives

$$\frac{L^2}{t^2} = \frac{Mg_{\text{local}}L}{m}$$

and

$$g_{\text{local}} = \frac{mL}{Mt^2} = \frac{(0.004 \text{ kg})(1.60 \text{ m})}{(3 \text{ kg})(0.0352 \text{ s})^2} = 1.722 \text{ m/s}^2$$

Pay attention to all units. 1 ms is 0.001m, 1g is 0.001 kg.

You can also find v and μ separately. And then find T from $v = \sqrt{\frac{T}{\mu}}$. And finally use $m \cdot g_{\text{local}} = T$ to find g .

ATTENTION: here T is tension force not period T . In order not to confuse you, you can always use F for this Tension in this kind of problem.

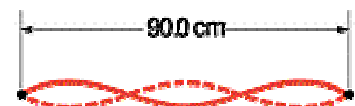
Problem 3, A nylon guitar string has a linear density of 7.5 g/m and is under a tension of 151 N . The fixed supports are 90 cm apart. The string is oscillating in the standing wave pattern shown below.

Calculate the

(a) speed, m/s

(b) wavelength, cm

(c) frequency of the traveling waves whose superposition gives this standing wave. Hz



Solution. (a) The wave speed is given by

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{(151 \text{ N})}{(0.0075 \text{ kg/m})}} = 141.9 \text{ m/s}$$

(b) The wavelength can be seen from the diagram to be equal to $2/3$ of the length of the string, or

$$\lambda = \frac{2}{3}L = \frac{2}{3}90 \text{ cm} = 60 \text{ cm} \quad (2L=3\lambda)$$

(c) The frequency can be calculated from the speed and the wavelength:

$$f = \frac{v}{\lambda} = \frac{(141.9 \text{ m/s})}{(0.6 \text{ m})} = 232 \text{ Hz}$$

A much short violin string will have a much higher fundamental frequency than this.

Problem 4. The organ pipe in the figure below is 1.5 m long.

(a) What is the frequency of the standing wave shown in the pipe?

Hz



(b) What is the fundamental frequency of this pipe? Hz

Solution. (a) For the pattern shown, the total length is equal to $3/4$ of a wavelength. $L = 3/4 \lambda$, So

$$\lambda = \frac{4}{3}L = \frac{4}{3}(1.5 \text{ m}) = 2.0 \text{ m}$$

and the frequency is

$$f = \frac{v}{\lambda} = \frac{(343 \text{ m/s})}{2.0 \text{ m}} = 171.5 \text{ Hz}$$

Notice that here the media for sound wave is not a string but air. As a result $v=343 \text{ m/s}$, unlike the previous questions .

(b) In the fundamental mode there is just $1/4$ of a wavelength in the pipe (see previous problem), and

$$\lambda = 4L = 4(1.5 \text{ m}) = 6.0 \text{ m} ,$$

$$f = \frac{v}{\lambda} = \frac{(343 \text{ m/s})}{6.0 \text{ m}} = 57.1 \text{ Hz}$$

Organ music has much lower tone comparing to short flute.