INTERFERENCE

This laboratory will investigate the phenomenon of interference. The interference and diffraction of light waves will be studied. Specifically, the interference patterns of a single slit, double slit, diffraction grating, and a human hair shaft will be analyzed.

I. Theory

Thomas Young's experiments with light in the early 19th century established it as having "wave-like" properties. At that time, results from experiments were suggesting that light could behave as a wave. Evidence seemed to be mounting that light is a wave, undermining Newton's contention that light consisted of particles. It was later discovered that light could behave as both a wave and a particle. The wave-particle duality of light can be experimentally observed, but both properties cannot be observed simultaneously. However, we have learned to live with the more profound and slightly uncomfortable idea of light being both a wave and a particle.

In Young's Double Slit experiment, coherent light shining through two slits gave rise not to the shadows of the slits, but to intensity patterns characteristic of wave interference. This pattern could be explained by associating light intensity with wave amplitude, that is light has "wave-like" properties. If any coherent light source is considered as a series of point sources arranged adjacent, then the light intensity at any location away from the source is determined by how waves coming from all points interfere as they arrive at that location.

The double-slit interference pattern consists of an array of bright bands (maxima, corresponding to constructive interference of light from the two slits), separated by dark bands (minima, where the interference is destructive). What is desired in this introduction section is the spacing of minima for each arrangement of slits. The spacing can be calculated by assuming some very familiar wave properties for light.

Calculating the spacing of minima for two infinitesimally narrow slits is fairly easy. It is found that for coherent light of wavelength $\lambda$ shining upon two infinitesimal slits, separated by a distance $d$, the light pattern cast on a screen a distance $D$ away is simply an infinite series of equally spaced dark and light bands of equal width and intensity. Actually, the intensity from each light to dark band varies smoothly between zero and a maximum value along the screen, but to the human eye, which has a threshold for intensities, the transition from light to dark appears sharp.

It can be shown that for two infinitesimal slits the maxima occur at positions along the screen determined by the following equation:

$$n\lambda = d\sin\theta$$
where $n$ is the number of maxima spots (or bands) away from the central maximum, $\lambda$ is the wavelength of coherent light used, $d$ is the slit spacing from center to center, and $\theta$ is the angle of deflection from the forward direction to the spot. The spacing, $\Delta x_{in}$, between adjacent interference minima is then found to be:

$$ \Delta x_{in} \cong \frac{\lambda D}{d} $$

Double slit interference for two infinitesimally narrow slits is illustrated in Figure 1, with $\Delta x_{in}$, $D$, and $d$ labeled and the insert showing the theoretical intensity variation along the screen. Any "real" slit has some finite width.

Light shining on a slit of finite width emerges from the slit as if it were emitted from a row of adjacent point sources which fill the slit. Thus an interference pattern is cast by a finite width single slit. The interference of light from a finite, "real" size, opening is called diffraction. For coherent light shining on a single slit of finite width, the pattern cast is a series of equally spaced bands, except for the central band which is twice as wide as the others. The central or center band in all the interference patterns is commonly called the "central maximum" or just "central
max". It can be shown that the minima cast by a finite width single slit occur along the screen when the following equation is satisfied:

\[ n\lambda = a\sin\theta \]

where \( n \) is the number of minima spots away from the central max, \( \lambda \) is the wavelength of coherent light used, \( a \) is the slit width, and \( \theta \) is the angle of deflection from the forward direction to the spot. The spacing between adjacent diffraction minima, \( \Delta x_{\text{diff}} \), is thus given by:

\[ \Delta x_{\text{diff}} \approx \frac{\lambda D}{a} \]

The interference (diffraction) pattern of a finite width, single slit is shown in Figure 2, with \( \Delta x_{\text{diff}} \), \( D \), and a labeled and the insert showing the theoretical intensity variation along the screen. The term "single slit diffraction" is somewhat misleading in that the pattern is not due to there being only one slit, but due to its finite width.

Figure 2. Interference pattern for a finite-width single slit.
The combined intensity patterns of a finite width single slit and two infinitesimal double slits can be observed in the interference (diffraction) pattern of two or more finite width double slits. This same interference pattern can be seen anytime a coherent beam of finite width passes by any type of "edge" and casts a shadow. The effect is to combine the patterns of Figure 1 and Figure 2, so that the resulting interference pattern is "modulated" by diffraction effects in each slit. This is shown in Figure 3, with $\Delta x_{\text{diff}}$, $\Delta x_{\text{int}}$, $D$, $d$, and $a$ labeled and the insert showing the theoretical intensity variation along the screen. Note from the equations that because $d > a$, $\Delta x_{\text{diff}}$ is the broad splitting and $\Delta x_{\text{int}}$ is the narrow one, or $\Delta x_{\text{diff}} > \Delta x_{\text{int}}$.

Now let's consider two special cases of single slit and double slit interference. A diffraction grating can be thought of as many infinitesimal double slits placed side by side. The interference pattern cast by a diffraction grating is a series of equally spaced "dots". Diffraction gratings produce maxima when the equation

$$n\lambda = ds\sin\theta$$

is satisfied, where $n$ is the order of the maximum spot, $\lambda$ is the wavelength of coherent light used, and $d$ is the slit or "line" spacing from center to center, and $\theta$ is the angle of deflection from the
forward direction to the dot. Note that this equation for the maxima of the diffraction grating is the same equation as for the maxima of the infinitesimal double slits. The spacing, $\Delta x_{\text{int}}$, between adjacent interference maxima is then given by:

$$\Delta x_{\text{int}} \cong \frac{\lambda D}{d}$$

where $D$ is the distance from the grating to the shadow screen.

Diffraction of light may be used to determine the thickness of a human hair. When a coherent beam of light is shined on a single strand of hair, the beam diffracts around the edges of the hair and casts an interference pattern. The interference pattern cast by the hair is very similar to that of a narrow single slit. Therefore we can treat the strand of hair as a single slit, except instead of the light going through the hair, it diffracts around the hair. The same equation describing diffraction minima for the single slit can be used to describe the diffraction maxima for a strand of human hair. Maxima for light diffracting around a strand of hair occur along the screen when the following equation is satisfied:

$$n\lambda = a\sin\theta$$

where $n$ is the order of the maxima, $\lambda$ is the wavelength of coherent light used, $a$ is the width of the hair shaft, and $\theta$ is the angle of deflection from the forward direction to the maximum. The spacing between adjacent diffraction maxima, $\Delta x_{\text{diff}}$, for the hair shaft is thus given by:

$$\Delta x_{\text{diff}} \cong \frac{\lambda D}{a}$$

where $D$ is the distance from the hair strand to the shadow screen.

**II. EXPERIMENTAL PROCEDURE**

**CAUTION! CAUTION! CAUTION!** DO NOT look directly into the laser beam! Damage to the retina can occur from exposure to the direct or reflected beam. The laser beam is harmless to any other part of the body. The power level of the laser used in this experiment is fairly safe, but staring down the beam can cause permanent loss of vision. The beam can be viewed without any concern when scattered from a diffuse surface like a piece of paper or the wall. Do not shine the laser beam across the room! BE CAREFUL!

**A. Determining the Width of a Single Slit Using Coherent Laser Light.**

1. Set up your optical bench with the laser about 25cm from the slide holder, pointed at the diffusing screen.
2. Choose the slide with several single slit patterns on it and place it in the slide holder. Adjust the position of the slide so that the laser beam is perpendicular to the slide. This is probably most easily accomplished by adjusting the slide so that the reflection of the laser beam back towards the laser goes back into the laser output aperture.
3. Shine the laser beam through each slit and select the slit that produces the clearest pattern. To obtain the clearest pattern, try moving the slide slightly from side to side. You should see a pattern of maxima and minima. It is easier to view this pattern from the downstream side of the screen.
4. Record the width of the particular slit you have chosen as given on the slide.
5. Place a piece of semi-transparent tape directly in the pattern of maxima and minima right on the screen.
6. Draw the pattern onto the tape you have placed on the screen. Affix the tape in your notebook as a record of your data.
7. Locate and mark the 6th (n=6) minimum on each side of the central maximum.
8. Measure and record the distance from the center of the central maximum to the center of each of the 6th minima. You should make two measurements of $x_6$ in this step.
9. Measure and record the distance from the slit you are using to the center of the central maximum and call this distance D.

Analysis/Discussion/Questions:
1. For this part of experiment,
\[ \sin \theta \equiv \tan \theta = \frac{x_6}{D} \]
so the equation describing minima becomes:
\[ 6\lambda = a\left(\frac{x_6}{D}\right) \]
Solving for $a$ we obtain:
\[ a = \frac{6\lambda D}{x_6} \]

2. Using the above expression for $a$, calculate two experimental values for $a$.
Note: $\lambda = 632.8$ nm; be sure to express all the variables in the same units.

3. Use the methods discussed in the lab on computer data analysis to calculate $\bar{a}$, $\sigma_a$, disc, # of sigmas, and Quality of Agreement. For the expected (or theoretical) value of $a$, use the value from the slit slide which you used.

B. Determining the Wavelength of a Laser Using Double Slit Interference (Diffraction).
Using the same setup as before, switch from the single slit slide to the double slit slide.
1. Shine the laser beam through each pair of slits and select the pair that produces the clearest pattern. To obtain the clearest pattern, try moving the slide slightly from side to side. You should see a pattern of maxima and minima.
2. Record the slit spacing for the slits you have chosen as given on the slide.
3. Place a piece of semitransparent tape directly in the pattern of maxima and minima right on the screen.
4. Draw the pattern onto the tape you have placed on the screen. Affix the tape in your notebook as a record of your data.
5. Locate the 8th (n=8) maximum on both sides of the central maximum.
6. Measure and record the distance $x_8$ from the center of the central maximum to the center of each of the 8th maxima. You should make two measurements in this step.
7. Measure and record the distance from the pair of slits you are using to the center of the central maximum and call this distance D.

Analysis/Discussion/Questions:
1. For this part of experiment as before,
\[
\sin \theta \equiv \tan \theta = \frac{x_8}{D}
\]
so the equation describing maxima becomes:
\[
8\lambda = d\left(\frac{x_8}{D}\right)
\]
Solving for \(\lambda\) we obtain:
\[
\lambda = \frac{dx_8}{8D}
\]

2. Using the above expression for \(\lambda\), calculate two experimental values for \(\lambda\). Note: Use the value for \(d\) that you recorded off of the slide and be sure to express all the variables in the same units.

3. Use the methods discussed in the lab on computer data analysis to calculate \(\bar{\lambda}\), \(\sigma_{\lambda}\), disc, # of sigmas, and Quality of Agreement. For the expected (or theoretical) value of \(\lambda\), use the expected value for a helium-neon laser.

Q1. Compare and contrast the double slit pattern with the single slit pattern.
Q2. Do you observe any single slit characteristics in the double slit pattern? Why or why not?

C. The Diffraction Grating.
Here we will observe diffraction by a diffraction grating. This is a series of many narrow slits, all with the same separation \(d\). The value of \(d\) for this grating is very small, about \(1.6 \times 10^{-6}\) m for a 600-lines-per-mm grating.

Put the grating on the holder in place of the slits and move the diffusing screen close to the grating. Observe the pattern on the diffusing screen. Describe it as carefully as you can in your lab book (diagrams are most useful), and then answer the following questions in your lab book.

Q3 How many spots do you see?
Q4. What is the maximum value of \(n\) for the spots you see? Why don't you see any more?
Q5. Describe how this pattern differs from the double-slit pattern that you observed earlier today?

For the following questions, refer to the diffraction grating relationship (which does not assume that the angle \(\theta\) is small):
\[
n\lambda = ds\sin \theta
\]

Q6. What features of the pattern are due to the fact that the slit spacing is very small?
Q7. Predict the angle from the central maximum to the first side spot. Does it agree roughly with what you see?
Q8. Why was the small-angle approximation good enough for the double-slit pattern but not for the pattern of the diffraction grating?
Q9. What would change about this pattern if the laser light were green instead of red?

D. Determining the Diameter of a Human Hair Using Interference of Laser Light.

In this part, you will determine the diameter of your hair strand by comparing its diffraction pattern to the diffraction pattern of a 100 μm slit. [See "Measuring the Diameter of a Hair by Diffraction" by S.M. Curry and A.L. Schawlow, American Journal of Physics, May 1974.]

Procedure/Data Taking:

1. Return the diffusing screen to the end of the optical bench.
2. Using masking tape, attach a single strand of your hair vertically across the slide holder. (Alternatively, you can just tape the hair directly to the laser, with the hair in the beam.)
3. Aim the laser beam at the strand of hair. You should see a pattern of maxima and minima. Adjust the position of the hair within the beam such that you obtain the clearest pattern.
4. Center the pattern of vertical lines shown in Figure 4 on the diffraction pattern produced on the screen by your hair strand. Remove Figure 4 from the lab manual by cutting it out and hold it up on the diffusing screen. (Just hold up this entire manual, if you don't want to cut anything out of it.)
5. Adjust the distance, D, between the hair strand and the manual until the pattern of minima produced by the hair strand lines up with the pattern of vertical lines in Figure 4.
6. Measure the distance from the hair strand to the screen and call this distance D.
7. The pattern lines in Figure 4 is chosen so that the distance D in centimeters is equal to the diameter of the hair in micrometers.
8. So, what is the diameter of your hair strand?

Q10. The equation nλ = a sinθ describes the positions of dark spots for a single slit, but of bright spots for the pattern produced by the hair. Can you think of a reason why this might be true?

IV. EQUIPMENT

Helium-neon laser with λ = 632.8 nm; beam should point towards the wall

Interf - 8
Optical bench
Single slit slide, double slit slide, and diffraction grating slide
Slide holder with stand
2-meter stick
Semi-transparent tape
Diffusing screen
Tensor lamp (for when the lights are off)