Physics 726, Problem set 6, Fall 2007

1. Obtain the Feynman rules for

$$\mathcal{L} = \frac{1}{2}(\partial^\mu \phi)(\partial_\mu \phi) - \frac{1}{2}m^2\phi^2 - g\phi^2(\partial^\mu \phi)(\partial_\mu \phi) \ .$$

When calculating the rule for the vertex, take all momenta ingoing. (Once you have the rule for the vertex, you can always change the sign on two momenta to make them outgoing.)

2. Consider the $SO(4)$ invariant lagrangian

$$\mathcal{L} = \frac{1}{2}\sum_{i=1}^{4}(\partial^\mu \phi_i)(\partial_\mu \phi_i) + \frac{1}{2}\mu^2 \sum_{i=1}^{4}\phi_i^2 - \frac{1}{4}\lambda\left(\sum_{i=1}^{4}\phi_i^2\right)^2 \ .$$

Take the vacuum expectation value to point in the ϕ_4 direction, and write $\phi_4 = v + \sigma$, with v the minimum of the potential energy.

(a) Find the amputated transition amplitude for $\sigma(k) + \phi_i(p_i) \to \sigma(k') + \phi_j(p_j)$ scattering, where the arguments of the fields denote the physical momenta of these particles. Amputating means that we omit the external lines, and the overall delta function for momentum conservation (bearing in mind, of course, that momentum is conserved). Write the amplitude in terms of the Mandelstam variables

$$s = (p_i + k)^2 \ , \qquad t = (p_i - p_j)^2 \ , \qquad u = (p_i - k')^2 \ .$$

(b) Prove that, in general, $s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2$, where $m_{1,2,3,4}$ are the four particles in the scattering process.

(c) Show that when the energy of the incoming and outgoing ϕ_i particles goes to zero, the amplitude goes to zero as well.