Physics 726, Problem set 2, Fall 2010

1. a) Take Maxwell’s equations in “covariant” form,
\[ \partial_\mu F^{\mu\nu} = J^\nu, \]
and show that they reduce to the standard inhomogeneous Maxwell equations if you use
\[ E^i = -E_i = F^{0i}, \quad F^{ij} = \epsilon_{ijk} B^k \]
with \( \epsilon_{123} = 1 \).

b) Show that
\[ \partial_\kappa F_{\mu\nu} + \partial_\mu F_{\nu\kappa} + \partial_\nu F_{\kappa\mu} = 0 \]
gives the homogeneous equations. [Reference: the final chapter of Griffiths: Introduction to Electrodynamics.]

2. Consider a boost in the direction of the \( z \)-axis,
\[ z' = \gamma(z - vt), \]
\[ t' = \gamma(t - vz), \]
with \( v \) the boost parameter, i.e. the relative velocity between the two inertial frames; we have set \( c = 1 \). In \( 2 \times 2 \) matrix form:
\[ \begin{pmatrix} z' \\ t' \end{pmatrix} = \begin{pmatrix} \gamma & -v\gamma \\ -v\gamma & \gamma \end{pmatrix} \begin{pmatrix} z \\ t \end{pmatrix}. \]
(Since \( x' = x \) and \( y' = y \) do not transform, we can simplify things by just considering \( 2 \times 2 \) matrices.)

a) Consider two successive boosts, one between the unprimed frame with coordinates \( x, y, z, t \) and the primed frame \( x', y', z', t' \) with boost parameter \( v \), and then one from the primed frame to a double-primed frame \( x'', y'', z'', t'' \) with boost parameter \( v' \). It is clear that the resulting transformation from the unprimed to the double-primed frame is again a boost in the \( z \) direction, with a boost parameter \( v'' \). Express \( v'' \) in terms of \( v \) and \( v' \), using the matrix form for a boost in the \( z \)-direction given above. We will call the boost that results from doing these two successive boosts the “product” of the boosts with parameters \( v \) and \( v' \). (But note that \( v'' \) is definitely not the product of \( v \) and \( v' \)!

b) Show that this set of transformations forms a group. In part a) you have shown that the product of two elements of this group is again an element of this group. So what remains is to show that there exists a unit element, that each element has an inverse, and that the associative law for the product defined in part a) holds.

c) The group we just studied is a subgroup of the full Lorentz group. Argue that it is sensible to call this subgroup \( \text{SO}(1,1) \). Show that this subgroup is abelian. (The full Lorentz group \( \text{SO}(3,1) \) is not!)

3. Study appendix 1 of chapter I.2 of Zee’s book. You do not have to hand in any work, but make sure you can do all the math in that appendix, and come ask me if you can’t!