Physics 726, Problem set 2, Fall 2007

1. a) Take Maxwell's equations in "covariant" form,

$$\partial_\mu F^{\mu\nu} = J^\nu \ ,$$

and show that they reduce to the standard inhomogeneous Maxwell equations if you use that $E^i = -E_i = F^{0i}$ and $F^{ij} = \epsilon_{ijk} B^k$ with ϵ_{ijk} the fully anti-symmetric tensor with $\epsilon_{123} = 1$.

b) Show that

$$\partial_\kappa F_{\mu\nu} + \partial_\mu F_{\nu\kappa} + \partial_\nu F_{\kappa\mu} = 0$$

gives the homogeneous equations.

2. a) Write the Feynman propagator as

$$\begin{aligned} D(x-y) &= -i \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} \left(\theta(x^0 - y^0) e^{-i\omega_k (x^0 - y^0)} + \theta(y^0 - x^0) e^{i\omega_k (x^0 - y^0)} \right) \\ &= -i\theta(x^0 - y^0) D_+(x-y) - i\theta(y^0 - x^0) D_-(x-y) \ . \end{aligned}$$

Prove that

$$D_\pm(x) = \frac{1}{(2\pi)^3} \int d^4k \delta(k^2 - m^2) \theta(\pm k^0) e^{-ikx} \ .$$

b) Argue that this implies that $D_\pm(x)$ is Lorentz invariant, *i.e.*, prove that $D_\pm(x) = D_\pm(\Lambda x)$, with Λ a Lorentz transformation. Do this by using that each of the elements, $\delta(k^2 - m^2)$, $\theta(\pm k^0)$, kx and d^4k are invariant. [Hint: use that $kx = (\Lambda^{-1}k)(\Lambda^{-1}x)$ for any Lorentz transformation Λ.]

3. In order to find the propagator for a massive spin-1 field, we need to invert the matrix $-(k^2 - m^2) g^{\mu\nu} + k^\mu k^\nu$. The inverse matrix has to be built from the four vector k (and the mass of the spin-1 field). Because of covariance under Lorentz transformations, the general form of the inverse has to look like

$$A(k^2) g_{\mu\nu} + B(k^2) k_\mu k_\nu \ .$$

Use this to find the inverse matrix, *i.e.*, determine the functions A and B.