problem set 2

1) Starting from the transformation law of the electromagnetic field-strength tensor $F^{\alpha\beta}$, derive the Lorentz transformation of the $\vec{E}$ and $\vec{B}$ fields under a boost in the $z$-direction. Use your results to describe what the electromagnetic field of a moving point charge looks like.

2) Show that Maxwell’s equations are invariant under gauge transformations on the vector potential $A_\alpha$

\[ \tilde{A}_\alpha = A_\alpha + \partial_\alpha \lambda, \]

in which $\lambda$ is some scalar field. This freedom of choice of the vector potential can be used to make it satisfy one condition, a so-called gauge condition. Show that if we choose this condition to be the Lorenz gauge $\partial_\alpha A^\alpha = 0$, it follows from Maxwell’s equations in empty space (no charges present) that the vector potential satisfies the wave equation.

3) Show that $F^{\alpha\beta}$ in empty space obeys the wave equation (irrespective of the choice of gauge!).

4) Consider a plane-wave solution to the wave equation, $A_\alpha = \epsilon_\alpha e^{ikx}$ with $kx = \vec{k} \cdot \vec{x} - \omega t$. Choose $A_\alpha$ to be in Lorentz gauge. Find $\vec{E}$ and $\vec{B}$ from the solution, and show that both $\vec{E}$ and $\vec{B}$ are transverse to the direction of propagation of the plane wave.