

Final, Physics 706, Spring 2011

This final is due on Thursday, May 19, at 2 pm in my mailbox, or with Caroline. There are three problems on this exam (two pages). The number of credit points is indicated for each problem. You are allowed to use your notes and the textbook by Sakurai and Napolitano *only* (the older edition is ok); do not consult with anyone; do not use Mathematica or similar. Good luck!

1. (17 points) An electron is at rest in an oscillating magnetic field pointing in the z direction, with magnitude $B_0 \cos \omega t$, in which B_0 and ω are constants. The relevant hamiltonian is

$$H = -\vec{\mu} \cdot \vec{B} = -\frac{e}{mc} \vec{S} \cdot \vec{B},$$

where m is the electron mass and e its charge.

- (a) (7 points) The electron starts out, at $t = 0$, in the spin-up state with respect to the x -axis. Determine the state at any subsequent time. [Hint: Watch out! The hamiltonian is time dependent, so you cannot get the state at time t from the stationary states! Solve the time-dependent Schrödinger equation directly.]
- (b) (5 points) Find the probability of getting $-\hbar/2$ if you measure S_x at time t .
- (c) (5 points) What is the minimum field B_0 required to force a complete flip of S_x ? *I.e.*, what is the minimum field B_0 required for there to exist a time at which $P(S_x = -\hbar/2) = 1$? If B_0 is larger than this minimum field, at what time does the first complete flip happen?

2. (16 points)

- (a) (8 points) A particle of spin 1 and a particle of spin 2 are at rest in a configuration such that the total spin is 3, and its total z -component is 1 (*i.e.*, the eigenvalue of S_z is \hbar). If you measured the z -component of the angular momentum of the spin-2 particle, what values might you get, and what is the probability of each one?
- (b) (8 points) An electron with spin up ($S_z = +\hbar/2$) is in the state $|n = 5, l = 1, m = 0\rangle$ of the hydrogen atom. If you could measure the total angular momentum of the electron alone (not including the proton spin), what values might you get, and what is the probability of each?

3. (17 points) Consider the one-dimensional harmonic oscillator, with angular frequency ω and mass m . Introduce the dimensionless coordinate $y = x(m\omega/\hbar)^{1/2}$.

(a) (4 points) Express y in terms of the raising and lowering operators a^\dagger and a , and use this to prove that

$$\langle m|y|n\rangle = \sqrt{\frac{n}{2}}\delta_{m,n-1} + \sqrt{\frac{m}{2}}\delta_{m,n+1} ,$$

where $|n\rangle$, $n = 0, 1, 2, \dots$, denote the energy eigenkets.

(b) (7 points)

Consider now the matrix elements

$$\langle m|y^3|0\rangle , \quad m = 0, 1, 2, \dots ,$$

Find all values of m for which these matrix elements do not vanish, and find the value of the nonvanishing matrix elements.

(c) (6 points) We perturb the oscillator by an additional potential $\delta V = \lambda\hbar\omega y^3$. Find the correction to the ground state energy to lowest nonvanishing order. (If you did not complete part (b), work on this part as much as you can.)