Problem Set #2

23.2 (a) by Newton's 3rd law \[ F = 1N \]
(b) since \( F = \frac{kq_2}{r^2} \) \( q_2 = 2q \), \( F = \frac{k(2q)q}{r^2} \) or \[ F = 2N \]
(c) now \( F = \frac{kq_2}{20r^2} \) so \[ F = 0.25N \]

23.15 \( q_1 = q_2 = -e \)
\( d = 10^{-10} \text{ m} \)
\( F_e = ? \)
\[ F_e = \frac{kq_1 q_2}{r^2} = \frac{ke^2}{r^2} = \frac{9 \times 10^9 \text{Nm}^2/\text{C}^2}{(10^{-10} \text{ m})^2} = 2 \times 10^{-6} \text{ N} \]

23.5 \( r = 1.0 \text{ mm} \)
\( Q = -1.0 \mu\text{C} \)
\( E(P) = ? \)
\[ E = \frac{kQ}{r^2} = \frac{9 \times 10^9 \text{Nm}^2/\text{C}^2}{(1 \times 10^{-3} \text{ m})^2} = 9 \times 10^9 \text{ N/C} \]
direction: Since the charge in question is negative, the electric field points along the line that connects points \( P \) and \( Q \), and away from point \( P \).

23.24 \( Q = -15 \mu\text{C} \) at \( (1.0 \text{ m}, 2.0 \text{ m}, 0) \)
\( P = (2.0 \text{ m}, -1.0 \text{ m}, 1.0 \text{ m}) \)
\[ \vec{r} = \vec{P} - \vec{Q} = (2-1) \hat{i} + (-1-2) \hat{j} + (1-0) \hat{k} \]
\[ = \hat{i} - 3 \hat{j} + \hat{k} \]
\[ |\vec{r}| = \sqrt{1^2 + 9^2 + 1^2} = \sqrt{81} = 9 \text{ m} \]
\[ \hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{1}{9} (\hat{i} - 3 \hat{j} + \hat{k}) \]
\[ \vec{E} = \frac{kQ}{r^2} \hat{r} = \frac{(9 \times 10^9 \text{Nm}^2/\text{C}^2)(-15 \times 10^{-6} \text{ C})}{11 \text{ m}^2} \frac{1}{9} (\hat{i} - 3 \hat{j} + \hat{k}) \]
\[ = -8.7 (\hat{i} - 3 \hat{j} + \hat{k}) = 8.7 (-\hat{i} + 3 \hat{j} - \hat{k}) \]
E-field points toward \( Q \).
23.7 \[ \overrightarrow{E} \text{ at } x = 1.0 \text{ m} \]

The only place where the fields can cancel is in the range PC (-1m, 1m).
Since the two charges are equal, the test charge \( q \) must be an equal distance between the two charges, at the origin.
Thus, \[
\overrightarrow{F} = \frac{kqQ}{x^2} \hat{i} + \frac{kqQ}{x^2} (-\hat{j}) = 0
\]

23.35

23.47 (a)
\[ Q = Q_2 = Q_3 = Q_4 = +Q \]
\[ E(P) = ? \]

since, \( E_1 = -E_4 \) and \( E_2 = -E_3 \),
\[ E_{\text{NET}} = 0 \]
All charges contribute

(b)
\[ E_1 = \frac{kQ}{a^2} \sin 45^\circ \hat{j} \]
\[ E_2 = \frac{kQ}{a^2} \hat{i} \]
\[ E_3 = \frac{kQ}{a^2} (\sin 45^\circ \hat{i} - \sin 45^\circ \hat{j}) \]
\[ = \frac{kQ}{2a^2} \sqrt{2} (\hat{i} - \hat{j}) \]

\[ E_{\text{NET}} = E_1 + E_2 + E_3 = \frac{kQ}{a^2} \left( 1 + \frac{1}{252} \right) (\hat{i} - \hat{j}) \]
\[ = 1.35 \frac{kQ}{a^2} (\hat{i} - \hat{j}) \]
Charge 4 does not contribute b/c it lies at point P.
$E_1 = \frac{kQ}{(5a/2)^2} (\sin \theta \hat{\imath} - \cos \theta \hat{\jmath})$

$E_3 = \frac{kQ}{(5a/2)^2} (\sin \theta \hat{\imath} + \cos \theta \hat{\jmath})$

$E_{\text{Net}} = E_{1x} + E_{3x} = 2 \frac{kQ}{5a/4} \sin(1.15) \hat{\jmath} = 1.46 \frac{kQ}{a^2} \hat{\jmath}$

all charges contribute

23.11 from Gauss law, $\Phi_E = \frac{Q_{\text{enc}}}{\varepsilon_0}$

so, for surface A, $Q_{\text{enc}} = 6Q + Q - Q = 6Q$,

$\Phi_E = \frac{6Q}{\varepsilon_0} = 3 \times 10^6 \text{ Nm}^2/\text{C}$

for surface B, $Q_{\text{enc}} = +6Q - 4Q = 2Q$,

$\Phi_E = \frac{2Q}{\varepsilon_0} = 1 \times 10^6 \text{ Nm}^2/\text{C}$

for surface C, $Q_{\text{enc}} = -3Q + Q - 2Q = 0$, $\Phi_E = 0$

23.16 (d) Since there are field lines present, there must be charge present, so (a) is false. By definition, when the net flux equals zero, every field line that exits the box must also enter. And by Gauss law, if the net flux is zero then the total charge inside must also be zero.
\[ \Delta \Phi_E = (\vec{E} \cdot \hat{n}) \Delta A \]

\[ l = 2.0 \text{m} \]
\[ \vec{E} = (15 \text{N/c}) \hat{i} + (27 \text{N/c}) \hat{j} + (39 \text{N/c}) \hat{k} \]
\[ A = l^2 = 4.0 \text{m}^2 \text{ for each face} \]

The right side, \( \hat{n} = \hat{i} \)
\[ \Delta \Phi_E = [(15 \text{N/c}) \hat{i} + (27 \text{N/c}) \hat{j} + (39 \text{N/c}) \hat{k}] \cdot \hat{i} (A) \]
\[ = (15 \text{N/c})(4.0 \text{m}^2) = 60 \text{Nm}^2/\text{c} \]

The left side, \( \hat{n} = -\hat{i} \)
\[ \Delta \Phi_E = [(15 \text{N/c}) \hat{i} + (27 \text{N/c}) \hat{j} + (39 \text{N/c}) \hat{k}] \cdot (-\hat{i}) A \]
\[ = -(15 \text{N/c})(4.0 \text{m}^2) = -60 \text{Nm}^2/\text{c} \]

The top side, \( \hat{n} = \hat{j} \)
\[ \Delta \Phi_E = [(15 \text{N/c}) \hat{i} + (27 \text{N/c}) \hat{j} + (39 \text{N/c}) \hat{k}] \cdot \hat{j} (A) \]
\[ = (27 \text{N/c})(4.0 \text{m}^2) = 108 \text{Nm}^2/\text{c} \]

The bottom side, \( \hat{n} = -\hat{j} \); \( \Delta \Phi_E \) is just the negative of the result for the top, so
\[ \Delta \Phi_E = -108 \text{Nm}^2/\text{c} \]

The front, \( \hat{n} = \hat{k} \)
\[ \Delta \Phi_E = [(15 \text{N/c}) \hat{i} + (27 \text{N/c}) \hat{j} + (39 \text{N/c}) \hat{k}] \cdot \hat{k} (A) \]
\[ = (39 \text{N/c})(4.0 \text{m}^2) = 156 \text{Nm}^2/\text{c} \]

The back, \( \hat{n} = -\hat{k} \); \( \Delta \Phi_E \) is just the negative of the result for the front, so
\[ \Delta \Phi_E = -156 \text{Nm}^2/\text{c} \]