Problem Set #1 - Math Review

1. Point P is located at \( x = 2, y = -1, z = 1 \). Point Q is located at \( x = 1, y = 2, z = 0 \). What is the vector that points from P to Q?

2. A train goes between three stops A, B, and C, in that order. From A to B, the train travels 75 km south and between B and C, it travels 55 km northwest.
   (a) Show the locations of A, B, and C on a diagram with clearly labeled coordinate axes.
   (b) Calculate the components of the train’s displacement between A and C.
   (c) What are the magnitude and direction of the displacement?

3. On your computer or graphing calculator, graph the function \( f(x) = x + 3\sin x \) in the rectangle \( x = [0,8] \) and \( y = [-1,8] \) (please, use radians).
   (a) Sketch the graph.
   (b) By visual inspection, on which interval is the average rate of change larger: \([1,2]\) or \([3,4]\)?
   (c) At which value of \( x \) is the instantaneous rate of change larger: \( x = 2 \) or \( x = 6 \)?
   (d) Check your visual estimates in part (b) by computing \( f'(x) \) and comparing the numerical values of \( f'(2) \) and \( f'(5) \).

4. A car moves on a straight line with an acceleration that increases linearly with time: \( a(t) = bt \), where \( b \) is a constant. Find the particle’s displacement and velocity as functions of time.

5. Consider the triangle below as a collection of horizontal strips of height \( dy \). Calculate the area of the triangle by integrating -- that is, by summing the areas of the differential strips. Note that the hardest part of this problem is the setup. For a refresher on integration techniques, refer to Lea and Burke, pages 248-253. Does your answer make sense?