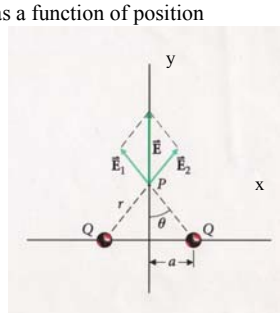
Calculation of E as *Function* of Position

- Last time, we calculated E at one specific location. Often we also want E as a function of position
- $E_1=E_2=kQ/(a^2+y^2)$
- $(E_1)_x= -(E_2)_x$
- $(E_1)_y = (E_2)_y = E_1\cos\theta$
- $\cos\theta = \dfrac{y}{\sqrt{a^2+y^2}}$
- $\vec{E} = 2E_1\cos\theta\hat{j}$
  $= \dfrac{2kQy}{(a^2+y^2)^{1.5}}\hat{j}$



Field lines vs. net charge in a volume

- Net # of electric field lines emerging from a volume is proportional to net charge inside volume
  - Positive # net field lines emerge: Q_{in} positive
  - Negative # net field lines emerge (i.e., more lines enter than leave): Q_{in} negative
  - Zero net field lines emerge: $Q_{in} = 0$

  (There might be charges inside, but total $Q_{in}=0$)
- This is a crude form of ***Gauss's Law***
- ***Question---***

Electric **Flux Φ_E**

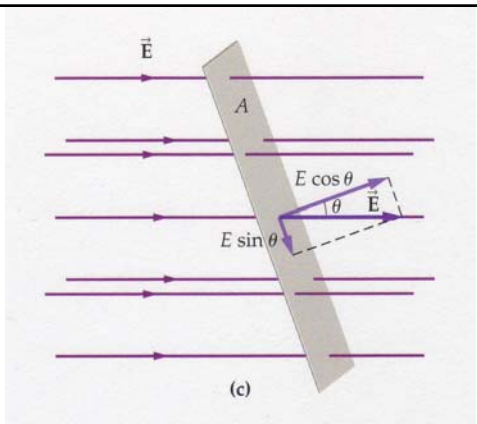
Φ_E is the amount of "flow" of E through a surface

- For a small element dA of area with normal $\hat{n}$ (a small enough dA that $\vec{E}$ is same everywhere on it)

$$d\Phi_E = (\vec{E} \bullet \hat{n})dA = E(\cos\theta)dA$$

where θ is the angle between $\vec{E}$ and $\hat{n}$

- $\hat{\mathbf{n}}$ is taken **outward** from a closed surface
- Unit of electric flux: $N\bullet m^2/C$



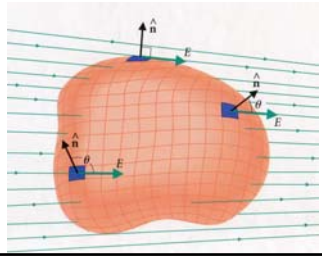
Electric Flux Example

- $\vec{E} = (2\hat{i} + 3\hat{j})$ N/C everywhere on a surface of area 2 m^2 which has the normal $(0.7\hat{i} - 0.7\hat{j})$. What is the electric flux through this surface?

Total electric flux out of closed surface

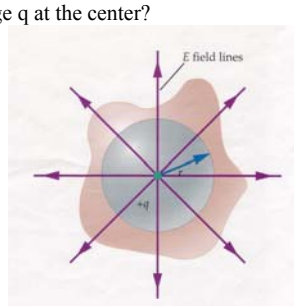
- ***Closed surface*** completely encloses a volume

$$\Phi_E = \oint d\Phi_E = \oint (\vec{E} \bullet \hat{n}) dA$$



Flux from point charge

- What is the electric flux out of a sphere of radius R from a charge q at the center?



Gauss's Law

- For the charge q at the center of the sphere, we found the electric flux out of the sphere = $4\pi kq$
- We get the same flux for ***any*** closed surface enclosing the charge, and the charge doesn't have to be at the center!
- Remember $k = 1/(4\pi\varepsilon_o)$, so we could write our result as $\Phi_E = q/\varepsilon_o = Q_{in}/\varepsilon_o$. This is Gauss's Law:

$$\Phi_E = \frac{Q_{in}}{\varepsilon_0}$$

where Φ_E is the electric flux out of a closed surface and Q_{in} is the net charge enclosed by the surface