

Write all vectors in terms of standard unit vectors i, j, k , and r as needed.

1. Identical point charges of magnitude q are placed at the 3 corners of an equilateral triangle of side a . What is the magnitude of the force on any one charge due to the other 2? (Circle the best answer)

- A. $2kq^2/a^2$
 B. kq^2/a^2
 C. $4kq^2/a^2$
 D. $3^{0.5}kq^2/a^2$
 E. $3^{0.5}kq^2/(2a^2)$



X-components of force cancel

Y-component: $2 \left(\frac{kq^2}{d^2} \right) \cos \theta$

$$\cos \theta = \frac{d}{a} = \frac{\sqrt{a^2 - a^2/4}}{a} = \frac{\sqrt{3}}{2} \Rightarrow F = \sqrt{3} kq^2/a^2$$

2. A thin sheet of charge measures 1.0 m wide by 2.0 m long. Which answer below is closest to the strength of the electric field at a position 0.010 m above the sheet of charge? (Circle the best answer)

- a) $0.0025 (C/m^2)/\epsilon_0$
 b) $0.005 (C/m^2)/\epsilon_0$
 c) $0.010 (C/m^2)/\epsilon_0$
 d) $0.50 (C/m^2)/\epsilon_0$
 e) $25 (C/m^2)/\epsilon_0$

$$Q = 0.010 C$$

$$E = \frac{\sigma}{2\epsilon_0} = \frac{Q}{2A\epsilon_0} \text{ from Gauss's Law (infinite sheet)}$$

$$= \frac{0.010 C}{2 \cdot 2m^2 \cdot \epsilon_0} = 0.0025 (C/m^2)/\epsilon_0$$

3. The electric field in a region is given by $E = (3 V/m^2) x i$ where x is the x -axis coordinate value. What is the value of the potential difference $V[x=3m, y=0] - V[x=2m, y=0]$? (Circle the best answer).

- a) 15V
 b) 7.5V
 c) 3V
 d) -3V
 e) -7.5V
 f) -15V

$$\Delta V = - \int_{x=2m, y=0}^{x=3m, y=0} \vec{E} \cdot d\vec{l} = - \int_{x=2m}^{x=3m} (3V/m^2) x dx = -3 \frac{V}{m^2} \left[\frac{x^2}{2} \right]_{2m}^{3m}$$

$$= -7.5V$$

4. The surface charge density is $2 C/m^2$ and the potential is 500V at point A on a conductor. What is the potential at point B on the same conductor if the surface charge density there is $1 C/m^2$? (Circle the best answer).

- a) 2000V
 b) 1000V
 c) 500V
 d) 250V
 e) 125V

Conductor is an equipotential!

5. In a region of space, a cylinder of length $L = 0.50$ m and ends of radius $R = 0.20$ m. There is no flux through the sides of the cylinder. There is uniform electric field of strength $E_1 = 4.0 \times 10^5$ N/C entering the cylinder through one end, and through the other end there is uniform electric field of strength $E_2 = 2.0 \times 10^5$ N/C exiting. The total charge, in μC , contained within the cylinder is: (Circle the best answer)

- A. -0.22
 B. 0.36
 C. -1.4
 D. +1.4
 E. -0.36

$$\Phi_E = \pi R^2 [2 \times 10^5 \text{ N/C} - 4 \times 10^5 \text{ N/C}] = \frac{Q_{in}}{\epsilon_0}$$

$$Q_{in} = \pi \epsilon_0 (0.2 \text{ m})^2 (-2 \times 10^5 \text{ N/C}) = -2.22 \times 10^{-7} \text{ C} = -0.22 \mu\text{C}$$

6. Charge is uniformly distributed throughout a long cylinder of radius 0.05 m. At a point a distance of 0.10 m from the center of the cylinder the electric field intensity is 8.9×10^5 N/C. The field, in N/C, at a distance of 0.04 m from the center of the cylinder is: (Circle the best answer)

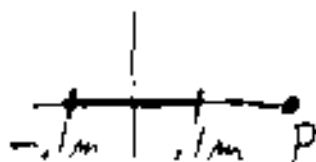
- A. 3.5×10^5
 B. 5.5×10^5
 C. 1.4×10^6
 D. 3.4×10^5
 E. 1.4×10^6

By Gauss's Law: $2\pi r_1 l E_1 = Q_{in} / \epsilon_0 = \frac{\rho \pi R^2 l}{\epsilon_0}$
 Thus $\rho = 2\epsilon_0 r_1 E_1 / R^2$

Then $2\pi r_2 l E_2 = \frac{\rho \pi r_2^2 l}{\epsilon_0} \Rightarrow E_2 = \frac{(2r_1 E_1 / R^2) r_2}{2} = \frac{(0.1 \text{ m})(0.04 \text{ m})}{(0.05 \text{ m})^2} E_1 = 1.4 \times 10^6 \text{ N/C}$

7. A charge of $1.3 \mu\text{C}$ is uniformly distributed along the x-axis from $x = -0.10$ m to $x = +0.10$ m. The potential, in V, at the point $x = 0.20$ m is: (Circle the best answer)

- A. 0.8×10^4
 B. 8.5×10^3
 C. 9.2×10^3
 D. 8.5×10^4
 E. 6.4×10^4



$$V = \int_{-0.1}^{+0.1} \frac{k(Q/l) dx}{(0.2 \text{ m} - x)} = -\frac{kQ}{l} \ln(0.2 \text{ m} - x) \Big|_{-0.1}^{+0.1}$$

8. A spherical conductor has a radius of $R = 0.15$ m. The potential at a distance of 0.45 m from the center of the sphere is measured to be 2.50×10^2 V. The surface charge density, in C/m^2 on the sphere is: (Circle the best answer)

- A. 4.4×10^{-8}
 B. 5.6×10^{-7}
 C. 5.6×10^{-6}
 D. 3.5×10^{-6}
 E. 3.5×10^{-5}

V at 0.45 m same as for pt. chg. at center

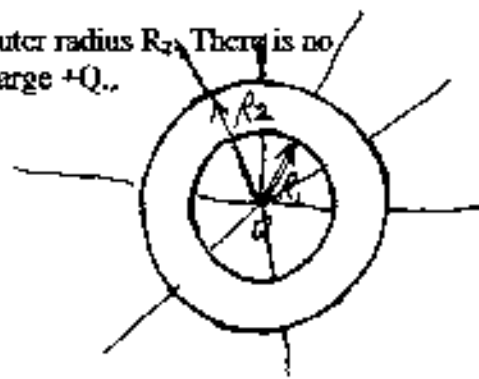
$$\frac{kQ}{0.45 \text{ m}} = 2.5 \times 10^2 \text{ V} \Rightarrow Q = 1.25 \times 10^{-8} \text{ C}$$

$$\sigma = \frac{Q}{A} = \frac{1.25 \times 10^{-8} \text{ C}}{4\pi (0.15 \text{ m})^2} = 4.4 \times 10^{-8} \text{ C/m}^2$$

Longer Problems. Show all work, and derive any needed relations from the "Starting Equations."

1. A conductor is in the shape of a spherical shell with inner radius R_1 and outer radius R_2 . There is no net charge on the conductor. At the center of the spherical shell is a point charge $+Q$.

(a) Draw a set of field lines on the diagram



(b) If r is the radial distance from the center, write down the electric field in the region $0 < r < R_1$ in terms of Q , r , R_1 , R_2 , the electric constants k and ϵ_0 , and appropriate unit vectors and standard constants as needed.

$$4\pi r^2 E = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{4\pi \epsilon_0 r^2}$$

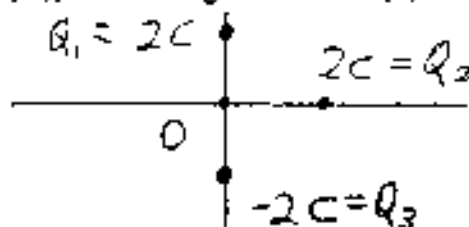
(c) Write down the electric field in the region $R_1 < r < R_2$

0 (conductor)

(d) Write down the electric field in the region $r > R_2$

$$4\pi r^2 E = \frac{Q}{\epsilon_0} \Rightarrow E = \frac{Q}{4\pi \epsilon_0 r^2}$$

2. Three charges are arranged as follows: A charge of $2C$ is at $(x, y) = (0, 1m)$; a charge of $2C$ is at $(1m, 0)$; and a charge $-2C$ is at $(0, -1m)$. What is the electric field at the origin?



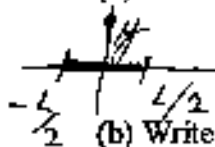
$$E_x = E_2 = \frac{kQ_2}{(1m)^2} = -1.8 \times 10^{10} \text{ N/C}$$

$$E_y = E_1 + E_3 = -2 \left(\frac{k \cdot Q_1}{(1m)^2} \right) = -3.6 \times 10^{10} \text{ N/C}$$

$$\vec{E} = -1.8 \times 10^{10} \frac{\text{N}}{\text{C}} \hat{i} - 3.6 \times 10^{10} \frac{\text{N}}{\text{C}} \hat{j}$$

3. A charge Q is uniformly distributed on a string which lies on the x -axis between $x = -L/2$ and $x = +L/2$. Give all answers in terms of L , Q , the electric constant k , and other standard constants such as π , as needed.

(a) What is the linear charge density of the string?



$$\lambda = \frac{Q}{L}$$

(b) Write down the charge element dQ you will use to find, by integration, the potential at position y on the y -axis. Give dQ in terms of dx , Q , L , x , y , and k as needed.

$$dQ = \lambda dx$$

(c) Write down the integral for the potential at position y on the y -axis, showing the upper and lower limits of integration.

$$V = \int_{x=-L/2}^{x=L/2} \frac{k \lambda dx}{(x^2 + y^2)^{3/2}}$$

(d) Carry out the integration to find the potential $V(y)$.