PHYS 230-01 Practice Exam on Chapters 30-33 Solutions

1. A circular loop of wire lies in the x-y plane and has an area of 2 m^2. A uniform but time-dependent magnetic field $B = (1.0)\ t^2\ T/s^2\ (2\hat{i} + 3\hat{k})$ exists in the region, where t is the time. What is the induced emf in the loop at time t= 4 s?

$$\mathcal{E}_{ind} = \frac{-d\Phi_B}{dt} = -\left[(2m^2\hat{k})(1T/s^2)(2t)\big|_{4s}\right]\cdot(2\hat{i}+3\hat{k})$$

$$= -48V$$

a) 32 V
b) 48 V
c) 64 V
d) 96 V
e) 160 V

2. A capacitor with an air core has plates of area 0.2 m^2 and a separation of 0.01 m. A displacement current of 3 μA flows between the plates. What is the rate of change of electric field between the plates?

$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 A \frac{dE}{dt}$$

$$\frac{dE}{dt} = \frac{I_d}{\varepsilon_0 A} = \frac{3\times10^{-6}A}{(0.2\,m^2)(8.85\times10^{-12}\,F/m)} = 1.7\times10^{6}\,V/m$$

a) 1.5×10^{-5} V/m/s
b) 7.5×10^{-5} V/m/s
c) 3.0×10^{-4} V/m/s
d) 1.7×10^{6} V/m/s
e) 3.4×10^{7} V/m/s

3. When a capacitor is discharged through a resistor, the time constant represents the time at which

a) The capacitor is completely discharged
b) The capacitor is 99% discharged
c) The capacitor is 50% discharged
d) The capacitor is 10% discharged
e) the charges on the capacitor is (1/e) of its original value

4. A solenoid has 1000 turns per meter and an area of 0.2 m^2 and carries a current of 3 mA. If the current is changed to 6 mA, how will the inductance of the solenoid change?

Inductance depends only on geometry of circuit

a) No change
b) Twice as large
c) Four times as large
d) Half as large
e) 1/4 as large

5. An inductor with L=0.042H is connected to a capacitor of 50 μF, which has an initial charge of 2.0×10^{-4} C, to form an LC circuit. The maximum energy stored in the magnetic field is

$$U_{B_{MAX}} = U_{E_{MAX}} = \frac{Q^2}{2C} = \frac{(2\times10^{-4}C)^2}{2(5\times10^{-5}F)}$$

$$= 4\times10^{-4}J$$

A. 2.0×10^{-4} J
B. 4.0×10^{-4} J
C. 8.0×10^{-4} J
D. 8.4×10^{-4} J
E. 1.6×10^{-3} J

6. The mutual inductance between two conducting loops is 15 mH. If the current through one coil changes at the rate of 2.4 A/s, the induced emf, in V, in the other coil is

A. 5.4×10^{-1}
B. 28
C. 6.25×10^{-2}
D. 3.6×10^{-2}
E. 6.6×10^{-1}

$$\mathcal{E}_{ind} = M\frac{dI}{dt} = (15\ mH)(2.4\ A/s) = 3.6 \times 10^{-2}\ V$$

7. A 400-mH inductor is connected to an AC power source of 60-V amplitude emf, at 40 Hz. The rms current through the inductor is

A. 1.1 A
B. 0.85 A
C. 0.42 A
D. 3.7 A
E. 0

$$X_L = \omega L = 2\pi(40/s)(0.4H)$$

$$I_0 = \frac{V_0}{X_L} = \frac{60V}{2\pi(40/s)(0.4H)} = 5.97 \times 10^{-1} A$$

$$I_{rms} = \frac{I_0}{\sqrt{2}} = 0.42\ A$$

8. Let R represent the resistance of an AC circuit, in series with a capacitive reactance of magnitude X_C and an inductive reactance of magnitude X_L. The impedance, Z, of the circuit is expressed

A. $R + X_L + X_C$
B. $\sqrt{R^2 + (X_L - X_C)^2}$
C. $R/(X_L + X_C)$
D. $R/\sqrt{X_L^2 - X_C^2}$
E. $\sqrt{R^2 + X_L^2 + X_C^2}$

$$\tilde{Z} = R + jX_L - jX_C$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

9. A variable frequency AC power source of 40 V (rms) is connected to an RLC circuit with R = 50 Ω, L = 20 mH, and C= 40 μF. The impedance of the circuit, in Ω, at 60 Hz is

A. 56
B. 7.6
C. 34
D. 77
E. 760

$$\omega = 2\pi(60/s)$$

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$= \sqrt{(50\Omega)^2 + (7.54\Omega - 66.3\Omega)^2} = 77\ \Omega$$

10. The current amplitude in the circuit at resonance is

A. 0.73 A
B. 0.39 A
C. 1.1 A
D. 2.3 A
E. 4.4 A

$$V_0 = \sqrt{2}\, V_{rms} = 56.6\ V$$

$$I_0 = \frac{V_0}{R} = \frac{56.6\ V}{50\ \Omega} = 1.1\ A$$

11. The polarization of an EM wave is taken to be
A. the direction of propagation of the wave.
B. the direction of the orientation of the electric field.
C. the direction of the orientation of the magneticfield.
D. the direction of the Poynting vector.
E. the direction halfway between the orientation of the electric field and that of the magnetic field.

12. In an electromagnetic wave propagating in a vacuum, the energy density produced by the electric field is
A. slightly larger than that created by the magnetic field.
B. slightly less than that created by the magnetic field.
C. significantly larger than that created by the magnetic field.
D. significantly less than that created by the magnetic field.
E. equal to that created by the magnetic field.

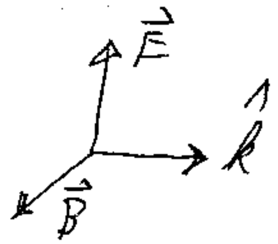
13. The light from a distant source has an electric field amplitude of 4.0 V/m. If the observer doubles the distance to the source, the electric field amplitude, in V/m, will be
A. 4.0
B. 8.0
C. 16.0
D. 2.0
E. 1.0

$$\frac{E_{02}^2}{2\mu_0 c} = I_{R_2} = \frac{P}{4\pi R_2^2} = \left(\frac{4\pi R_1^2}{4\pi R_2^2}\right) I_{R_1} = \frac{1}{4} I_{R_1} = \frac{1}{4}\frac{E_{01}^2}{2\mu_0 c}$$

$$\Rightarrow E_0^2 = \frac{E_{01}}{2} = 2.0\ V/m$$

14. In a plane electromagnetic wave propagating in the +z direction, with unit vector $\hat{k}$, the electric field $\vec{E}$, the magnetic induction $\vec{B}$, and $\hat{k}$ must satisfy

A. EXB=0, kXB=E
B. E•B= 0, kXE=B
C. E•B= 0, kXB=E
D. EXB=0, kXE=B
E. E•B = 0, k•E = 0



15. The intensity of an EM wave, in a vacuum, is 4.2 X 10^{-4} S1 units. The electric field amplitude, in SI units, of the wave is
A. 8.8 X 10^{-3}
B. 3.5X 10^{-2}
C. 0.56
D. 1.8 X 10^{2}
E. 7.3 X 10^{3}

$$I = \frac{E_0^2}{2\mu_0 c} \quad = 0.56\ V/m$$

$$E_0 = \sqrt{2\mu_0 c I} = \sqrt{2(4\pi\times10^{-7}\ N/A^2)(3\times10^8\ m/s)(4.2\times10^{-4}\ W/m^2)}$$

16. A plane electromagnetic wave propagates through a vacuum with frequency 1.40 X 10^{15} Hz. The amplitude of the electric field strength is 8.50 V/m. The wavelength of the radiation, in nm, is

A. 31.0
B. 214
C 817
D. 439
E. 9.00

$$\lambda f = c$$

$$\lambda = \frac{c}{f} = \frac{3\times10^8\ m/s}{1.4\times10^{15}/s} = 214\ nm$$

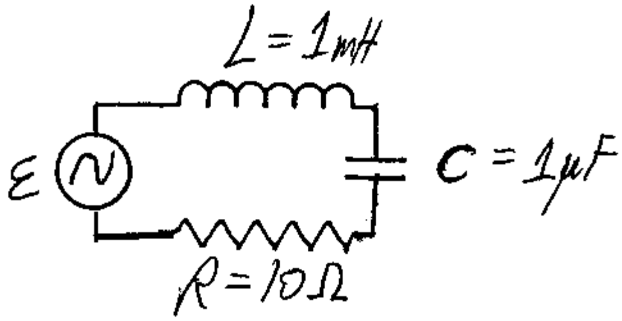
17. The (time-averaged) amplitude of the Poynting vector, in SI units, for the above wave is

A. 1.81×10^{-1}
B. 6.50×10^{-3}
C. 2.33
D. 3.45×10^{2}
E. 9.58×10^{-2}

$$\langle S \rangle = \frac{E_0^2}{2\mu_0 c} = \frac{(8.5\ V/m)^2}{2(4\pi \times 10^{-7}\ N/A^2)(3\times 10^8\ m/s)} = 9.58\times 10^{-2}\ W/m^2$$

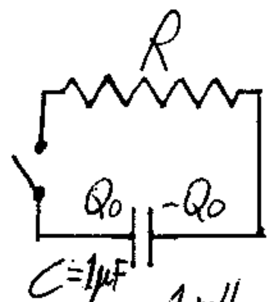
PROBLEMS

1. Consider the circuit shown to the right. Assume the EMF is given by $\varepsilon = V_0 \cos(\omega t)$, with $V_0 = 10$ V.



a) What is the impedance, Z, of the circuit at a frequency f = 6000 Hz?
b) What is the current amplitude in the circuit, I_0, for a drive frequency of 6000 Hz?
c) Determine the amplitude of the voltage drop across the resistor, the capacitor, and the inductor (f=6000 Hz).
d) Draw a voltage phasor diagram for the circuit at 6000 Hz.
e) What frequency would produce the largest current amplitude?

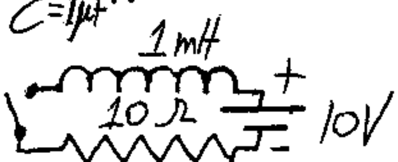
2 . Consider the circuit shown to the right. When the switch is closed at t = 0 the charge on the capacitor is Q_0.

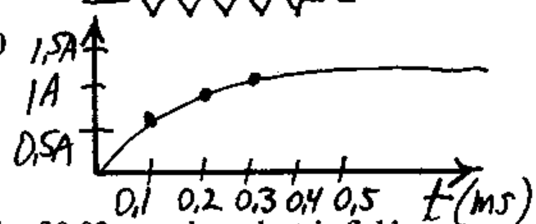


(a) Describe what will happen in the circuit.

(b) What resistance is required for the charge on the capacitor to fall to half its initial value in 1 msec (0.001 sec)?

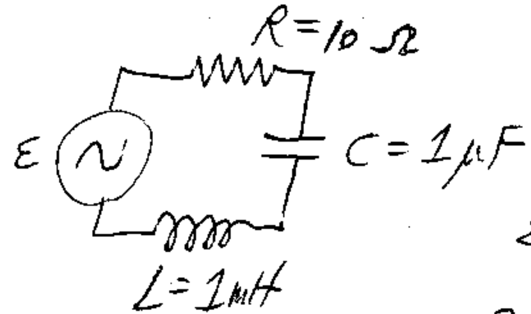
3. Consider the circuit shown to the right. The switch is closed at t = 0. (a) Draw a graph of the current in the circuit for times from t=0 to t=0.5 ms on the given axes.
(b) What is the current a long time after the switch is closed? $\boxed{1A}$
(c) What is the voltage drop across the inductor at t = 0.1 ms? $\boxed{3.7V}$





4. An EM wave travelling in the x-direction through the vacuum of space has a wavelength of 0.02 m and an electric field amplitude $E_0 = 2.00 \times 10^{-3}$ V/m.
a) Write complete expressions for the magnitudes of the wave's E and B fields as a function of time.
b) The wave is absorbed by a 10 m^2 surface that is oriented normal to the direction the wave is traveling. How much energy is absorbed by the surface in one minute?
c) What is the average force exerted on the surface by the wave?
d) The surface is removed and the wave, which is initially unpolarized, is allowed to pass through two polarizers whose transmission axes are offset by 20° relative to each other. What is the amplitude of the E field that emerges from the second polarizer?

1.



a) $\tilde{Z} = R + j(\omega L - 1/\omega C)$

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\omega = 2\pi (6000/s) = 3.77 \times 10^4 \text{ rad/s}$$

$$\omega L = 37.7\,\Omega$$

$$\left(\frac{1}{\omega C}\right) = 26.5\,\Omega$$

$$Z = \boxed{15.0\,\Omega}$$

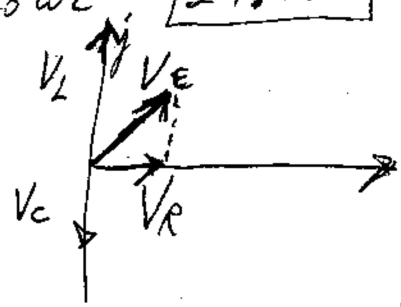
b) $I_0 = \frac{V_0}{Z} = \frac{10V}{15\Omega} = \boxed{0.66A}$

c) $V_R = I_0 R = \boxed{6.6V}$

$V_C = I_0 \left(\frac{1}{\omega C}\right) = \boxed{17.5V}$

$V_L = I_0 \omega L = \boxed{24.9V}$

d)



e) $f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi(3.16 \times 10^{-5} s)} = \boxed{5.04 \times 10^3 \text{ Hz}}$

2. a) Capacitor charge will decay exponentially. Current will decay.

b) Need $Q_0 e^{-1ms/RC} = 0.5 Q_0$

$$-1ms/RC = \ln(0.5)$$

$$R = \frac{(-1ms)}{(\ln(0.5))(10^{-6}F)} = \boxed{1.44 \times 10^3\,\Omega}$$

3. a) $\tau = \frac{L}{R} = \frac{.1\,mH}{10\,\Omega} = 0.1\,ms$

$$I = I_0\left(1 - e^{-t/(L/R)}\right)$$

$t = \tau;\ I = I_0(1 - .37) = .63\,A$

$t = 2\tau;\ I = I_0(1 - .135) = .86\,A$

$t = 3\tau;\ I = I_0(1 - .05)\quad .95\,A$ See plot

b) $I_0 = 10V/10\Omega = \boxed{1A}$

c)
$$\frac{dI}{dt} = +I_0\left(\frac{R}{L}\right)e^{-1}$$

$$\mathcal{E}_L = -L\frac{dI}{dt} = 0.37 I_0 R = \boxed{3.7V}$$

4.
a) $\vec{E} = \mathbf{E}_0 \cos(kx - wt)$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.02m} = 3.14 \times 10^2 m^{-1}$

$w = ck = (3\times 10^8\,m/s)(3.14\times 10^2 m^{-1}) = 9.42 \times 10^{10}\,Rad/s$

$E_0 = 2.00 \times 10^{-3} V/m \qquad B_0 = \frac{E_0}{c} = 6.67 \times 10^{-12}\,T$

$$\boxed{\begin{aligned} E &= (2.00\times 10^{-3} V/m)\cos[(314\,m^{-1})x - (9.42\times 10^{10}/s)t] \\ B &= (6.67\times 10^{-12}\,T)\cos[(314\,m^{-1})x - (9.42\times 10^{10}/s)t] \end{aligned}}$$

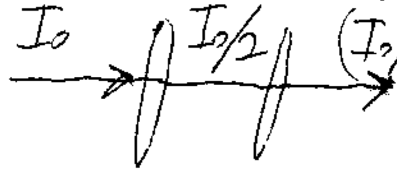
b) $U = IAt = \langle S\rangle At = \frac{E_0^2 At}{2\mu_0 c} = \boxed{3.18 \times 10^{-6} J}$

4. (c) $F = \frac{dp}{dt} = A\frac{\langle S \rangle}{c} = A\frac{E_0^2}{2\mu_0 c^2}$

$$= \frac{(10\,m^2)(2\times10^{-3}\,V/m)^2}{2(4\pi\times10^{-7}\,N/A^2)(3\times10^8\,\frac{m}{s})^2}$$

$$= \boxed{1.77\times10^{-16}\,N}$$

(d) I_0 $\quad I_1 = I_0/2 \quad I_2 = (I_0/2)\cos^2\theta$



$$E_1 = \frac{E_0}{\sqrt{2}}$$

$$E_2 = E_1\cos\theta = \frac{E_0}{\sqrt{2}}\cos\theta$$

$$= \frac{(2\times10^{-3}\,V/m)}{\sqrt{2}}\cos(20^\circ)$$

$$= \boxed{1.33\times10^{-3}\,V/m}$$