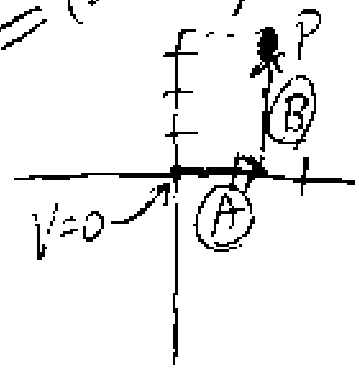
PHYS 230

SOLUTIONS - PROB. SET 4

1. (25-29)

$$\vec{E} = \left(16\,\tfrac{V}{m}\right)\hat{i} + \left(8.5\,\tfrac{V}{m}\right)\hat{j}$$



$$V(P) = -\int_{0,0}^{x=1.5\,m,\ y=3.5\,m} \vec{E}\cdot d\vec{l}$$

$$= \underbrace{-\int_{x=0,\ y=0}^{x=1.5\,m,\ y=0} \left[\left(16\,\tfrac{V}{m}\right)\hat{i} + \left(8.5\,\tfrac{V}{m}\right)\hat{j}\right]\cdot\left(dx\,\hat{i}\right)}_{\text{path } A}$$

$$-\int_{x=1.5\,m,\ y=0}^{x=1.5\,m,\ y=3.5\,m} \left[\left(16\,\tfrac{V}{m}\right)\hat{i} + \left(8.5\,\tfrac{V}{m}\right)\hat{j}\right]\cdot\left(dy\,\hat{j}\right)$$

$$= -16\,\tfrac{V}{m}\left(1.5\,m - 0\right) - 8.5\,\tfrac{V}{m}\left(3.5\,m - 0\right)$$

$$V(P) = \boxed{-54\,V}$$

2. (25-34)

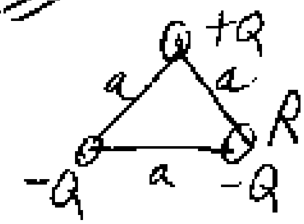
$$\Delta V = -\int_{x=1.5\,cm}^{2.8\,cm} \vec{E}\cdot d\vec{l} = -\int_{x=1.5\,cm}^{2.8\,cm} \frac{E_0\,x\,dx}{a}$$

$$d\vec{l} = dx\,\hat{i}$$

$$= \left[-\frac{E_0 x^2}{2a}\right]_{0.015\,m}^{0.028\,m} =$$

$$= -\frac{5.0\times10^3\,V/m}{2(0.01\,m)}\left[(.028\,m)^2 - (.015\,m)^2\right] = \boxed{-140\,V}$$

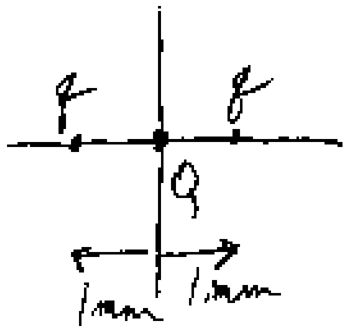
3. (25-42)



By superposition

$$V(R) = \frac{kQ}{a} + \frac{k(-Q)}{a} = \boxed{0}$$

4. (25-44)



$$U = \sum_{\text{all pairs}} \frac{kQ_1Q_2}{d}$$

$$= k\left[\frac{(3\times10^{-6}C)(10^{-6}C)}{10^{-3}m} + \frac{3\times10^{-12}C^2}{10^{-3}m} + \frac{(10^{-6}C)^2}{2\times10^{-3}m}\right]$$

$$= \boxed{+58J}$$

If each q is released, each will gain $K = \frac{U}{2} = 29J$

Thus $\frac{1}{2}mV_f^2 = \frac{U}{2} = 29J$

$$V_f = \sqrt{\frac{2\cdot(29J)}{0.01kg}} = \boxed{76\ m/s}$$

5. (25-8) The field has the largest magnitude in the region where the potential changes most per unit distance, which is at $\boxed{\text{position (d)}}$

The field points from higher to lower potential, or from $\boxed{e \to a}$.

6. (25-9)

The charge Q is all at the same distance a from the center, so treat as set of point charges:

$$V = \int \frac{k\,dQ}{r} = \frac{k}{a}\int dQ = \boxed{\frac{kQ}{a}}$$

7. (25-11)

Answer $\boxed{(c)}$. The field is zero everywhere within the conductor!

8. (25-50)

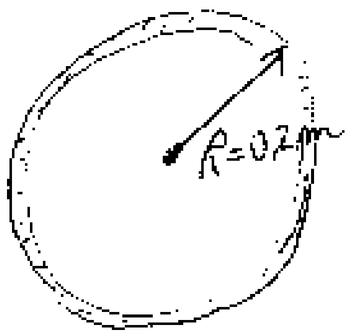
The magnitude of the field is $E \approx \frac{\Delta V}{\ell}$

$$E_A \cong \frac{10^{-3}V}{0.5\,m} = \boxed{2\times 10^{-3}\ V/m}$$

$$E_B \cong \frac{10^{-3}V}{0.7\,m} = \boxed{1.4\times 10^{-3}\ V/m}$$

Both point $\boxed{\text{toward origin}}$ (credit for 1-2×1

9. (25–60)



$Q = 1.0\,\mu C$

Since there is no charge at $r < R$ and the $\vec{E}$ field must be spherically symmetric, Gauss's Law ~~tells~~ tells us $E = 0$ for $r < R$, and the potential is the <u>same</u> for all $r < R$. For $r \geq R$, Gauss's Law gives

$$\Phi_E = 4\pi r^2 E = \frac{Q}{\varepsilon_0}$$

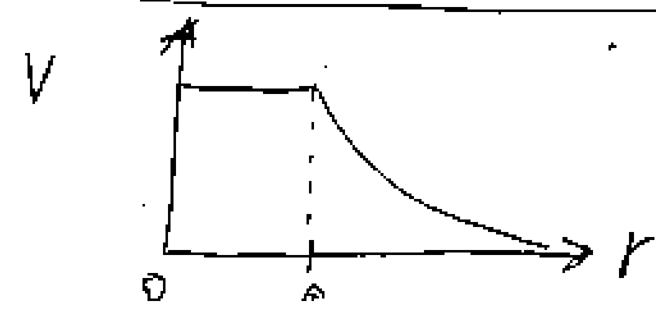
$$E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{kQ}{r^2} \text{ radially outward,}$$

This is the field of a point charge at the center, so the potential will be $V(r \geq R) = \dfrac{kQ}{r}$

$$= \frac{\left(9\times10^{9}\,\frac{N\cdot m^2}{C^2}\right)\left(1\times10^{-6}\,C\right)}{r}$$

So
$$\boxed{V(r \geq R) = \frac{9\times10^{3}\,V\cdot m}{r}}$$

and
$$\boxed{V(r<R) = V(R) = \frac{9\times10^{3}\,V\cdot m}{0.2\,m} = 4.5\times10^{4}\,V}$$



10. (25-78) For conductors $E = \frac{\sigma}{\varepsilon_0}$ at surface

Thus $\sigma = \varepsilon_0 E = (8.85 \times 10^{-12}\, C^2/Nm^2)(1.57 \times 10^5\, V/m)$

$|\sigma| = 1.4 \times 10^{-6}\, C/m^2$