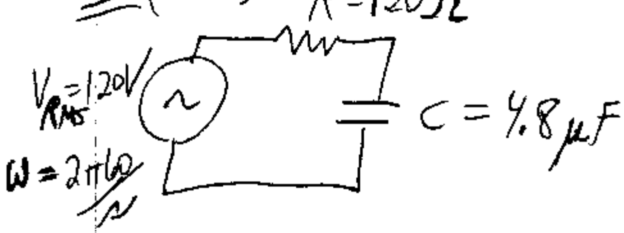
SOLUTIONS - PROB. SET 13

1. (32-9)

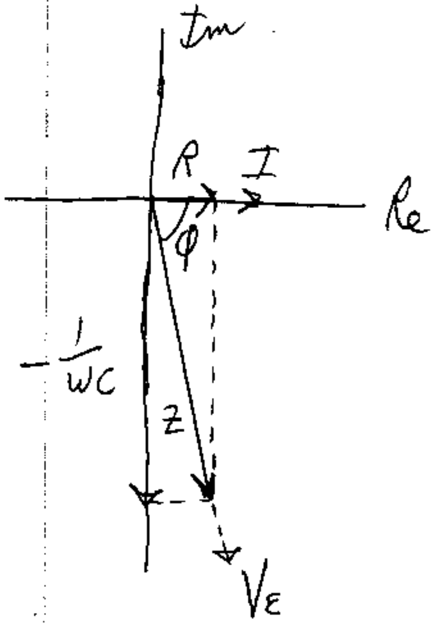


$$\tilde{Z} = R - \frac{j}{\omega C}$$

$$Z = \sqrt{R^2 + \left(\frac{-1}{\omega C}\right)^2}$$

$$= \sqrt{(120\,\Omega)^2 + \left(\frac{1}{(377/s)\cdot 4.8\,\mu F}\right)^2}$$

$$= 566\,\Omega$$



$$\phi = \tan^{-1}\left(-\frac{1/\omega C}{R}\right)$$

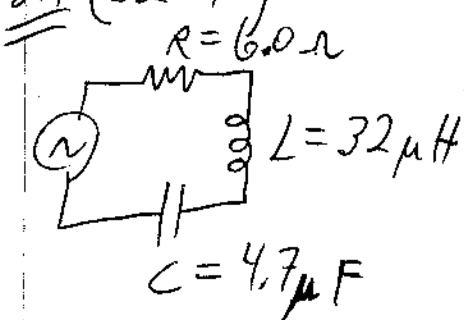
$$= \tan^{-1}\left(-\frac{553\,\Omega}{120\,\Omega}\right) = -1.36 \text{ rad}$$

$$\langle P \rangle = \frac{V_{RMS}^2}{Z}\cos\phi$$

$$= \frac{(120V)^2}{566\,\Omega}\cos(-1.36 \text{ rad})$$

$$= \boxed{5.4W}$$

2. (32-10)



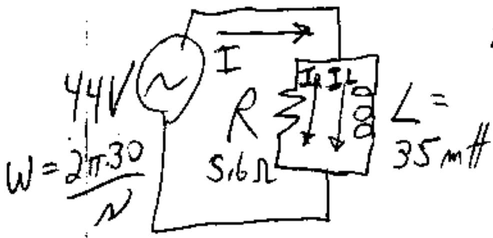
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(32\,\mu H)(4.7\,\mu F)}}$$

$$= 8.15 \times 10^4/s$$

$$f_0 = \frac{\omega_0}{2\pi} = \boxed{13 \text{ kHz}}$$

$$Q = \frac{\omega_0 L}{R} = \frac{(8.15\times 10^4/s)(32\,\mu H)}{6.0\,\Omega} = \boxed{0.43}$$

3. (32-42)



$$\frac{1}{\tilde{Z}_p} = \frac{1}{R} + \frac{1}{j\omega L} = \frac{j\omega L + R}{j\omega L R}$$

$$\tilde{Z}_p = \frac{j\omega L R}{R + j\omega L} = \frac{\omega^2 L^2 R + j(\omega L R^2)}{R^2 + \omega^2 L^2}$$

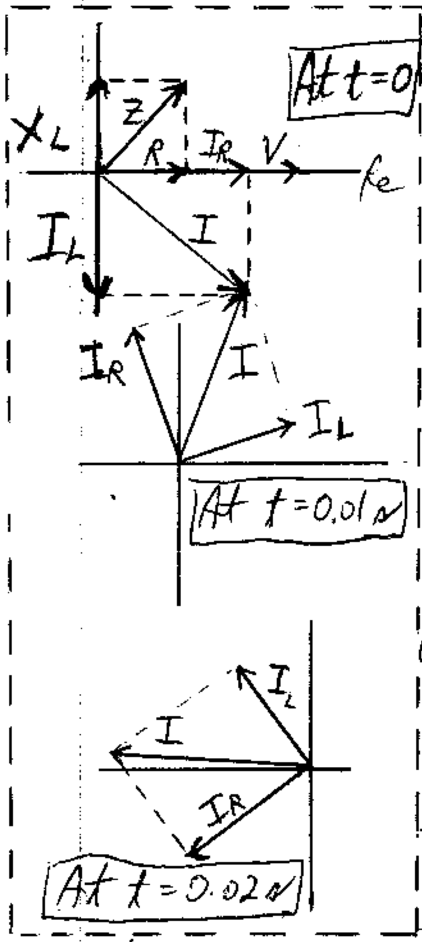
$$\left|\frac{1}{Z_p}\right| = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{\omega L}\right)^2} =$$

$$= \sqrt{\left(\frac{1}{5.6\,\Omega}\right)^2 + \left(\frac{1}{\underbrace{(189/s)(35\,mH)}_{6.6\,\Omega}}\right)^2}$$

$$= 0.234\,\Omega^{-1}$$

$$Z_p = 4.27\,\Omega$$

$$I_0 = \frac{V_0}{Z} = \frac{\sqrt{2} \cdot 44V}{4.27\,\Omega} = \boxed{15\ A}$$



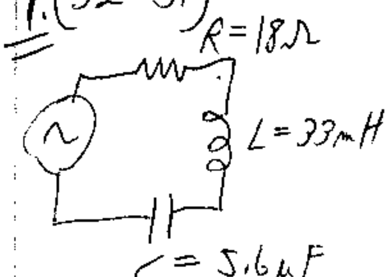
Extra credit Phasor Diagrams

$$\phi = \tan^{-1}\left(\frac{\omega L R^2}{\omega^2 L^2 R}\right) = \tan^{-1}\left(\frac{R}{\omega L}\right)$$

$$= \tan^{-1}\left(\frac{5.6\,\Omega}{6.6\,\Omega}\right)$$

$$= \boxed{0.70\ rad}$$

4. (32-51)



$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(33\,mH)(5.6\,\mu F)}}$$

$$= 2.33 \times 10^3\ rad/s$$

$$f_0 = \frac{\omega_0}{2\pi} = \boxed{370\ Hz}$$

$$Q = \frac{\omega_0 L}{R} = \frac{(2.33 \times 10^3/s)(33\,mH)}{18\,\Omega} = \boxed{4.3}$$

5. (33-4)

$$\vec{B} = (4.3 \times 10^{-10}\,T)\,\hat{z}\cos\left\{\left[\frac{\pi}{(1.0\,mm)}\right](x - ct)\right\}$$

We find E_0 from $B_0 = \frac{E_0}{c} \Rightarrow E_0 = cB_0$

$$= (3 \times 10^{8}\,m/s)(4.3 \times 10^{-10}\,T)$$

$$= 1.3 \times 10^{-1}\,V/m$$

The wave is propagating along $\hat{x}$ since it depends on $(x - ct)$. Thus $\hat{k} = \hat{x}$ and $\vec{B} = B\hat{z}$ so we need $\vec{E} = E\hat{y}$ to get $\vec{k} \parallel \vec{E} \times \vec{B}$.

Thus $\boxed{\vec{E} = \left(0.13\,\frac{V}{m}\right)\hat{y}\cos\left\{\left(\frac{\pi}{1.0\,mm}\right)(x - ct)\right\}}$.

6. (33-6) We have $\langle|\vec{S}|\rangle = \frac{\langle|\vec{E} \times \vec{B}|\rangle}{\mu_0} = \frac{\langle|\vec{E}|\cdot|\vec{B}|\rangle}{\mu_0}$

$$= \frac{E_0^2}{c\mu_0}\langle\cos^2(kx - \omega t)\rangle$$

$$= \frac{E_0^2}{2c\mu_0}$$

But also $\langle S\rangle \cdot \left[\pi\left(\frac{d}{2}\right)^2\right] = 1.5 \times 10^{-3}\,W$

Thus $d^2 = \left(\frac{4}{\pi}\right)\left(\frac{2c\mu_0}{E_0^2}\right)(1.5 \times 10^{-3}\,W) = 5.76 \times 10^{-6}\,m^2$

E_0 = (500 V/m)

$\boxed{d = 2.4 \times 10^{-3}\,m}$

23058-15-4

7. (33–19) $f = 1.65\times10^7\ \mathrm{Hz}$, propagating along $\hat{x}$

$E_0 = 0.792\ \mathrm{V/m}$

Thus from $\frac{\omega}{k} = c$ we have $k = \frac{\omega}{c} = \frac{2\pi f}{c}$

$$= \frac{2\pi\cdot(1.65\times10^7/\mathrm{s})}{3.0\times10^8\ \mathrm{m/s}}$$

$$= 0.346\ \mathrm{m}^{-1}$$

Also $B_0 = \frac{E_0}{c} = \frac{0.792\ \mathrm{V/m}}{3.0\times10^8\ \mathrm{m/s}}$

$$\boxed{\begin{aligned} B_0 &= 2.64\times10^{-9}\ \mathrm{T} \\ \vec{k} &= 0.346\ \mathrm{m}^{-1}\ \hat{x} \end{aligned}}$$

8. (33–33) The energy flux from the Sun is

$\langle S\rangle = 1.4\times10^3\ \mathrm{W/m^2}$ (page 1045)

We need $0.055\langle S\rangle A = 350\ \mathrm{W}$

$$A = \frac{350\ \mathrm{W}}{(0.055)\cdot(1.4\times10^3\ \mathrm{W/m^2})}$$

$$= \boxed{4.5\ \mathrm{m}^2}$$