PH 230-01

SOLUTIONS - PROB. SET 2

1. (23-56) Field lines leave a closed box, but every line that leaves the box reenters.

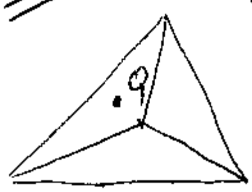
a) False – there must be charge in the box or else no field lines would emerge.

b) False - there would have to be a net flux out of the box if it held a net positive charge

c) False – there would have to be a net flux into the box if it held a net negative charge.

d) True

2. (23-57) By Gauss's law $\Phi_E = \dfrac{Q_{in}}{\varepsilon_0}$



$$= \frac{6.5 \times 10^{-8}\,C}{8.85 \times 10^{-12}\,C^2/N\cdot m^2}$$

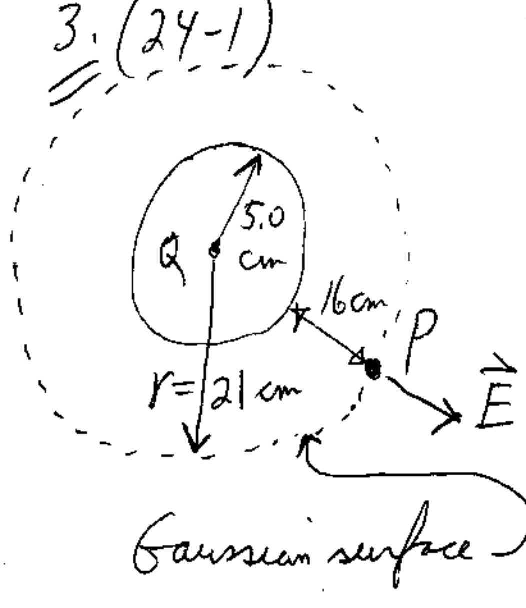
$$= \boxed{7300 \frac{N\cdot m^2}{C}}$$

To find the flux through each face, we would need to know the location of the charges.

If the charge were at the center the flux would be ~~the~~ same through each face

$$\Phi_{E_{face}} = \frac{\Phi_E}{4} = 1800 \frac{N\cdot m^2}{C}$$

3. (24-1)



To find the field $\vec{E}$ at P we use Gauss's Law with a Gaussian surface consisting of a sphere of radius 21 cm. as shown.

Then, since $\vec{E} = E\hat{r}$ and $\hat{n} = \hat{r}$ we have $\vec{E}\cdot\vec{n} = E$ and

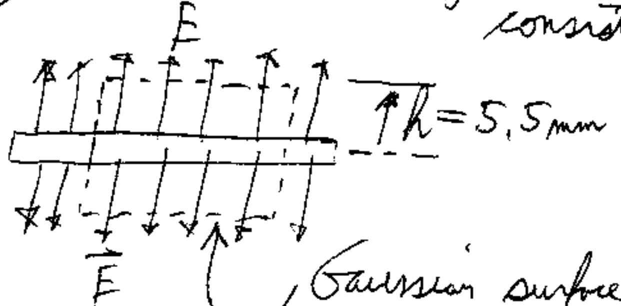
$$\Phi_E = \oint \vec{E}\cdot\vec{n}\,dA = E\oint dA = 4\pi r^2 E = \frac{Q_{in}}{\varepsilon_0}$$

Thus $$E = \frac{Q_{in}}{4\pi\varepsilon_0 r^2} = \frac{16\times10^{-9}\,C}{4\pi\left(8.85\times10^{-12}\frac{C^2}{N\cdot m^2}\right)(0.21\,m)^2}$$

$$= 3300\,\frac{N}{C}$$

So $\vec{E} = \boxed{3300\,\frac{N}{C}\,\hat{r}}$ (Note typo error in text answer)

4. (24-5) Apply Gauss's Law with a Gaussian surface consisting of a box of height $2h$ and top & bottom area each $= A$.



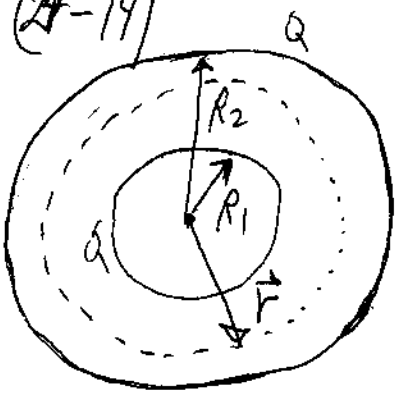
Then $\Phi_E = EA + EA = \frac{Q_{in}}{\varepsilon_0}$

So $2EA = \frac{\sigma A}{\varepsilon_0}$

$$\vec{E} = \frac{(4.6\times10^{-3}\,C/m^2)}{2(8.85\times10^{-12}\,C^2/N\cdot m^2)}\,\hat{j} = \boxed{2.6\times10^{8}\,N/C\,\hat{j}}$$

5. (24-12) Can use Gauss's Law for points near the centers of the faces. For points near edges, we do not have enough symmetry to make Gauss's Law useful.

6. (24-14)



Apply Gauss's Law with a spherical Gaussian surface of radius $r > R_1$ and $r < R_2$. The field must be radial by symmetry, $\vec{E} = E\hat{r}$, so

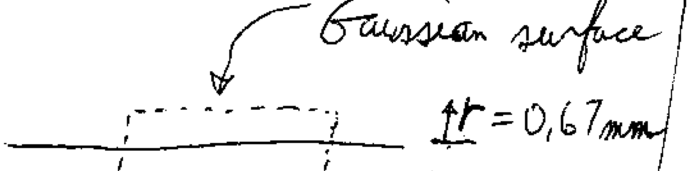
$$\Phi_E = \underbrace{\oint}_{\text{Gaussian surface}} \vec{E}\cdot\hat{r}\,dA = E\int dA = \frac{Q}{\varepsilon_0}$$

Thus $4\pi r^2 E = \dfrac{Q}{\varepsilon_0}$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r} = \boxed{\frac{kQ\hat{r}}{r^2}}$$

7. (24-20)

Apply Gauss's Law with cylindrical surface of radius $r = 0.67\,\text{mm}$ and length l, $\vec{E} = E\hat{r}$ and $\hat{n} = \hat{r}$, so



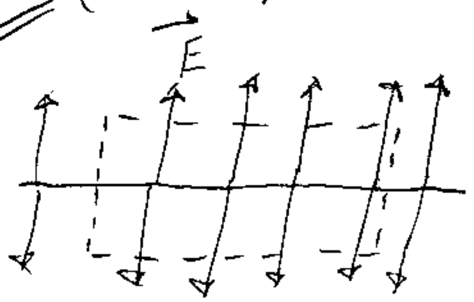
$$\oint \vec{E}\cdot\hat{n}\,dA = 2\pi r l E = \frac{Q_{in}}{\varepsilon_0} = \lambda l/\varepsilon_0$$

Thus $\vec{E} = \dfrac{\lambda}{2\pi\varepsilon_0 r}\hat{r}$

$$= \frac{-2.7\times10^{-9}\,\text{C/m}\,\hat{r}}{2\pi\,(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)\,(6.7\times10^{-4}\,\text{m})}$$

24-20 continued: Thus $\boxed{\vec{E} = -7.2 \times 10^4 \frac{N}{C} \hat{r}}$

8. (24-22)



As in previous problem

$$E = \frac{\lambda}{2\pi \varepsilon_0 r}$$

Thus $\lambda = 2\pi \varepsilon_0 r E$

$$= 2\pi \left(8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2}\right)\left(1.2 \times 10^{-2} m\right) \cdot \left(1.5 \times 10^4 \frac{N}{C}\right)$$

$$\lambda = \boxed{1.0 \times 10^{-8}\ C/m}$$

The filament has length $l = 15$ cm, so its charge is

$$Q = \lambda l = \left(1.0 \times 10^{-8} \frac{C}{m}\right)\left(1.5 \times 10^{-1} m\right)$$

$$= \boxed{+1.5 \times 10^{-9}\ C}$$

(The filament must have positive charge since electrons are attracted toward it.)