15-2 Pressure

Pressure is force per unit area:

- **Definition of Pressure,** \( P \)
  \[ P = \frac{F}{A} \]

- SI unit: \( \text{N/m}^2 \) or **Pascal (Pa)**

1 Pa = 1 N/m²

Other common pressure units:
- Pounds per Square Inch (PSI) - tires, etc.
- mm Hg - blood pressure
- inches Hg - weather barometer

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Pressure vs. Contact Area

The same force applied over a smaller area results in greater pressure – think of poking a balloon with your finger and then with a needle.

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Example

- What is pressure exerted by 80-kg person’s shoe on floor if shoe is 0.3 m by 0.1 m and each shoe supports half of total weight?
  \[
  P = \frac{W/2}{A} = \frac{(80\text{kg})(9.8\text{N/kg})/2}{(0.3\text{m})(0.1\text{m})} = 1.3 \times 10^4 \text{N/m}^2 = 13 \text{ kPa}
  \]

- What if half of weight rests on 0.005m by 0.005m stiletto heel?
  \[ P = 1.6 \times 10^7 \text{ Pa} \]
Measuring Pressure

Instruments for pressure measurement:
- manometer
- barometer
- pressure gauge

This is a mercury manometer - pressure $P_0$ lifts column of mercury to height $h$.
Can give pressure reading in “mm of Hg”

Blood Pressure Readings

- The higher (systolic) number represents pressure while the heart contracts to pump blood to the body.
- The lower (diastolic) number represents pressure when the heart relaxes between beats.
- The systolic pressure is always stated first. For example: 118/76 (118 over 76); systolic = 118, diastolic = 76. Blood pressure below 120 over 80 mmHg (millimeters of mercury) is considered optimal for adults.

Atmospheric Pressure

Atmospheric pressure is due to the weight of the atmosphere above us.

$$P_{at} = 1.01 \times 10^5 \text{ Pa} = 101 \text{ kPa}$$

This is standard atmospheric pressure at sea level. It declines with altitude above sea level.

<table>
<thead>
<tr>
<th>Location</th>
<th>$P_{at}$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>San Francisco</td>
<td>101</td>
</tr>
<tr>
<td>Denver, Co</td>
<td>84</td>
</tr>
<tr>
<td>Mt. Whitney</td>
<td>60</td>
</tr>
<tr>
<td>Mt. Everest</td>
<td>35</td>
</tr>
</tbody>
</table>
Atmospheric Pressure
There are a number of different ways to describe atmospheric pressure.

In pascals: \[ P_{at} = 101 \text{ kPa} \]
In pounds per square inch: \[ P_{at} = 14.7 \text{ lb/in}^2 \]
In bars: \[ 1 \text{ bar} = 10^5 \text{ Pa} \approx 1 P_{at} \]
In inches of mercury: 29.9 in Hg

Weather Report Barometer
- Apparent 52°
- Dew Point 44°
- Humidity 76%
- Wind S/5 mph
- Visibility 10 mi
- Barometer 30.03 in Hg (slightly high)

Sensitivity to Pressure
- Atmospheric pressure does not crush us, as our cells maintain an internal pressure that balances it.
- Since atmospheric pressure acts uniformly in all directions, we don’t usually notice it.
- We notice when internal and external pressures are different - pressure difference on ear when descending in airplane, etc.

Gauge Pressure
Most pressure gauges measure amount of pressure above atmospheric pressure - this is called gauge pressure.

If you want to add air to your tires to manufacturer’s specification, you are not interested in total pressure. You are interested in gauge pressure – how much more pressure is there in tire than in atmosphere?

\[ P_g = P - P_{at} \]
Example - Tire Pressure Gauge

Expert Quality, handy-size tyre pressure gauge - range 2.0-99.5 PSI (15-700 kPa)
Reads gauge pressure of air in tires. Actual pressure would be \( P_g + P_{\text{atm}} \).
Typical tires: \( P_g = 30 \text{ PSI} = 205 \text{ kPa} \)
\[
P = 44.7 \text{ PSI} = 305 \text{ kPa}
\]

Pressure in Fluids

Pressure is the same in every direction in a fluid at a given depth; if it were not, the fluid would flow.

Pressure versus Depth in Fluid

There is an increase in fluid pressure as we descend further below top surface. This is due to increasing weight of fluid above us.
For an open container of cross-sectional area \( A \) and height \( h \) filled with fluid of density \( \rho \)
\[
F_{\text{top}} = P_{\text{at}} A
\]
\[
F_{\text{bottom}} = P_{\text{at}} A + \rho Ahg
\]
\[
P_{\text{bottom}} = \frac{F_{\text{bottom}}}{A}
\]
\[
P_{\text{bottom}} = P_{\text{at}} + \rho gh
\]

Pressure vs. Depth

The same relation works between any two points separated by vertical distance \( h \).

Dependence of Pressure on Depth

\[
P_2 = P_1 + \rho gh
\]

This relation is valid for any liquid whose density does not change with depth.
Example: Find pressure at 100 m below ocean surface.

\[ P = P_{atm} + \rho_{W}gh \]
\[ P = 101 \text{ kPa} + (10^3 \text{ kg/m}^3)(9.8 \text{ N/kg})(100\text{m}) \]
\[ = 1081 \text{ kPa} \text{ (about 10 times } P_{atm}) \]

Pressure and Depth

Pressure-depth relation is true in any container where the fluid can flow freely – the pressure at the same height will be the same everywhere.

“Fluid Seeks It's Own Level”

Question

- Three containers a), b), c) each have the same bottom surface area, \( A \), and are each filled with water to the same height \( h \).
- Pressure as a function of depth = \( P_{atm} + \rho_{W}gh \).

Which container has the greatest pressure of water pushing on the bottom surface?

Pascal’s Principle

If an external pressure is applied to a confined fluid, pressure at every point within the fluid increases by that amount. This principle is used, for example, in hydraulic lifts:

\[ F_{out} = F_{in} \]
\[ P_{out} = P_{in} \]
\[ A_{out} = A_{in} \]
**Pascal’s Principle: Work-Energy**

Force $F_1$ needed on piston of area $A_1 = 0.04 \text{m}^2$ to lift 5000N car on piston of area $A_2 = 4 \text{m}^2$?

Need pressure $P_2 = \frac{5000 \text{N}}{4 \text{m}^2} = P_1 = \frac{F_1}{0.04 \text{m}^2}$

Thus $F_1 = \frac{0.04 \text{m}^2}{4 \text{m}^2}(5000 \text{N}) = 50 \text{N}$

**What distance $d_1$ does $F_1$ need to push down to raise car a distance $d_2 = 0.1 \text{m}$?**

Volume of fluid displaced on left, $V = d_1 A_1$, equals volume increase on right, $V = d_2 A_2$. So, $d_1 = \frac{d_2 A_2}{A_2} = 10 \text{m}$.

Work done by $F_1$ is $W_1 = F_1 d_1$ is same as work done by $F_2$:

$W_2 = F_2 d_2 = \frac{d_2 (A_2 F_1/A_1)}{(d_2 A_2)/(F/A_1)}$

$= \frac{d_1 A_1 (F_1/A_1)}{F_1 d_1} = W_1$

Energy Conservation - Small force $F_1$ pushing through a large displacement $d_1$ is converted to large force $F_2$ pushing through small displacement $d_2$.

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**Pascal’s Principle in Hydraulic Brakes**

A fluid exerts a net upward force on any object it surrounds, called the buoyant force.

This force is due to the increased pressure at the bottom of the object compared to the top.

$$F_b = F_2 - F_1 = \rho g L^3$$
Buoyancy and Archimedes’ Principle

There is a net force on an object submerged in a fluid because the pressures at the top and bottom of the object are different.

The buoyant force is found to be the upward force on the same volume of water:

\[ F_B = F_2 - F_1 = \rho_f g A (h_2 - h_1) \]

\[ = \rho_f g A \Delta h \]

\[ = \rho_f V g \]

where \( V \) is object volume.

Archimedes’ Principle

Object completely immersed in a fluid experiences upward buoyant force equal in magnitude to weight of fluid displaced by object.

Buoyant Force When a Volume \( V \) Is Submerged in a Fluid of Density \( \rho_f \)

\[ F_B = \rho_f V g \]

SI unit: N

- Buoyant force is the same for a totally submerged object of any size, shape, or density
- The buoyant force is exerted by the fluid
- Whether an object sinks or floats depends on relationship between buoyant force and weight

Applications of Archimedes’ Principle

An object floats when it displaces an amount of fluid equal to its weight.

Some fraction of object volume will be submerged.

Iceberg: \( \rho(\text{ice}) < \rho(\text{water}) \)

An Iceberg is floating in the ocean. As the iceberg melts, does the ocean level 1) rise, 2) sink, or 3) stay the same.

As ice, the iceberg displaces a volume of water equal in mass to the iceberg’s mass. Once it is melted, the iceberg still displaces a volume of water equal in mass to the original iceberg (melting doesn’t change the iceberg mass).
Archimedes’ Principle: Totally Submerged Object

- The upward buoyant force is \( F_b = \rho_{\text{fluid}} g V_{\text{obj}} \)
- The downward gravitational force is \( W = mg = \rho_{\text{obj}} g V_{\text{obj}} \)
- The net force is \( F_b - W = (\rho_{\text{fluid}} - \rho_{\text{obj}}) g V_{\text{obj}} \)

Depending on the direction of the net force, the object will either float up or sink!

Archimedes’ Principle: Floating Object

For a floating object, the fraction that is submerged depends on the densities of the object and of the fluid.

\[
F_B = \rho_f V_{\text{displ}} g \\
mg = \rho_{\text{obj}} V_{\text{obj}} g
\]

Submerged Volume of Floating Object

- Object of volume \( V_{\text{obj}} \) and density \( \rho_{\text{obj}} \) floating in fluid of density \( \rho_f \).
- Weight of object \( W_{\text{obj}} = \rho_{\text{obj}} V_{\text{obj}} g \) must equal \( F_b = \rho_f V_{\text{sub}} g \).
- Thus:

\[
V_{\text{sub}} = V_{\text{obj}} \left( \frac{\rho_{\text{obj}}}{\rho_f} \right)
\]

- For example, object with density 20% that of water will be only 20% submerged.
Applications of Archimedes’ Principle

An object made of material that is denser than water can float only if it has indentations or pockets of air that make its average density less than that of water.

Buoyancy and Archimedes’ Principle

If an object of density less than that of water is placed under water, there will be an upward net force on it, and it will rise until it is partially out of the water.

This principle also works in the air; this is why hot-air and helium balloons rise.

End of Lecture 26

- For Monday, Nov. 9, read Walker 15.5-9.
- Homework Assignment 15a is due at 11:00 PM on Tuesday, Nov. 10.