Energy Conservation in Oscillatory Motion

In an ideal system with no nonconservative forces, total mechanical energy is conserved. For a mass on a spring:

\[ E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \]

Since we know the position and velocity as functions of time, we can find the maximum kinetic and potential energies:

\[ U_{\text{max}} = \frac{1}{2}kA^2 \]
\[ K_{\text{max}} = \frac{1}{2}mA^2\omega^2 = \frac{1}{2}mA^2(k/m) = \frac{1}{2}kA^2 \]

Energy Conservation in Oscillations

As a function of time,

\[ E = U + K = \frac{1}{2}kA^2 \cos^2(\omega t) + \frac{1}{2}kA^2 \sin^2(\omega t) \]
\[ = \frac{1}{2}kA^2[\cos^2(\omega t) + \sin^2(\omega t)] = \frac{1}{2}kA^2 \]

Total energy is constant; as kinetic energy increases, potential energy decreases, & vice versa.

\[ E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = \frac{1}{2}mv_{\text{max}}^2 = \text{constant} \]

\[
\begin{array}{c}
\text{anywhere} \\
\downarrow \\
\text{at } x = \pm A \\
\downarrow \\
\text{center}
\end{array}
\]

Energy Conservation in Oscillatory Motion

This diagram shows how the energy transforms from potential to kinetic and back, while the total energy remains the same.
Using Conservation of Energy
A 0.5 kg block on spring with k = 100 N/m is pulled a distance of 0.2 m from equilibrium and released. How fast is it going when it gets to equilibrium position?

At x=A=0.2m, energy all potential:
E = \frac{1}{2}kx^2 = \frac{1}{2}(100 \text{ N/m})(0.2 \text{ m})^2 = 2 \text{ J}

At equilibrium position (x=0) energy all kinetic:
E = 2J = \frac{1}{2}mv^2; \; v^2 = 4J/0.5kg = 8 \text{ m}^2/\text{s}^2
v = 2.83 \text{ m/s} = v_{\text{max}}

Using Conservation of Energy
What is the speed of the block when x = 0.1m?

\[ E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = \frac{1}{2}kA^2 - kx^2 \]
\[ mv^2 = kA^2 - kx^2 \]
\[ v = \sqrt{\frac{k}{m} \left( A^2 - x^2 \right)} \]
\[ = \sqrt{\frac{100 \text{ N/m}}{0.5 \text{ kg}}} \left[ (0.2 \text{ m})^2 - (0.1 \text{ m})^2 \right] \]
\[ = 2.45 \text{ m/s} \]

Question 1
Four springs have been compressed from their equilibrium positions at x = 0. Which system has the largest maximum speed in its oscillation?

(a) \hspace{1cm} (b) \hspace{1cm} (c) \hspace{1cm} (d)

The Simple Pendulum
A simple pendulum consists of a mass m (of negligible size) suspended by a string of length L (and negligible mass).

The angle \theta that it makes with the vertical approximately does SHM.
**Period of Simple Pendulum**

The period of a simple pendulum depends only on $g$ and the length $L$ of the string (and is independent of mass):

$$ T = 2\pi \sqrt{\frac{L}{g}} $$

**Damped Oscillations**

In most oscillations, there are nonconservative forces such as air drag which tend to decrease the amplitude of the oscillation over time.

**Critical Damping**

If you don’t want oscillation of a mass-spring system, add enough drag to get critical damping.

Then, the system returns to equilibrium as fast as possible without oscillating.

Used in car shock absorbers.

**Driven Oscillations & Resonance**

An oscillation can be driven by an oscillating driving force; the frequency of the driving force may or may not be the same as the natural frequency $f_0$ of the system.

Pendulum: $f_0 = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{g}{L}}$

Mass+Spring: $f_0 = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$
Driven Oscillations & Resonance

If the driving frequency is close to the natural frequency, the amplitude can become quite large, especially if the damping is small. This is called resonance.

Small damping: sharp peak
Large damping: broad peak

The resonant frequency $f_0$ is not shifted by damping.

Fluid Mechanics (Chap. 15)

- Fluid - Material with no definite shape; takes shape of container
  - Liquid
  - Gas
- Can move by "flow"
- Properties:
  - Density
  - Pressure
  - Buoyant Force
  - Volume flow rate
  - Viscosity (fluid friction)

15-1 Density

The (mass) density of a material is its mass $M$ per unit volume $V$:

\[
\rho = \frac{M}{V}
\]

SI unit: kg/m$^3$

The densities of most liquids and solids vary slightly with changes in temperature and pressure.

Densities of gases vary greatly with changes in temperature and pressure.

Density Values

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (kg/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water (at 4°C)</td>
<td>1000 (1 g/cm$^3$)</td>
</tr>
<tr>
<td>Lead</td>
<td>11,300</td>
</tr>
<tr>
<td>Gold</td>
<td>19,300</td>
</tr>
<tr>
<td>Air</td>
<td>1.29</td>
</tr>
<tr>
<td>Helium</td>
<td>0.18</td>
</tr>
</tbody>
</table>

The specific gravity of a substance is the ratio of its density to that of water.
15-2 Pressure
Pressure is force per unit area:

Definition of Pressure, $P$

$P = F/A$

SI unit: N/m$^2$ or Pascal (Pa)

1 Pa = 1 N/m$^2$

Other common pressure units:
- Pounds per Square inch (PSI) - tires, etc.
- mm Hg - blood pressure
- inches Hg - weather barometer

Pressure vs. Contact Area
The same force applied over a smaller area results in greater pressure – think of poking a balloon with your finger and then with a needle.

Example
- What is pressure exerted by 80-kg person’s shoe on floor if shoe is 0.3 m by 0.1 m and each shoe supports half of total weight?

$P = \frac{W/2}{A} = \frac{(80 \text{ kg})(9.8 \text{ N/kg})/2}{(0.3 \text{ m})(0.1 \text{ m})} = 1.3 \times 10^4 \text{ N/m}^2$

= 13 kPa

- What if half of weight rests on 0.005 m by 0.005 m stiletto heel?

$P = 1.6 \times 10^7 \text{ Pa}$

Measuring Pressure
Instruments for pressure measurement: manometer, barometer, pressure gauge

This is a mercury manometer - pressure $P_0$ lifts column of mercury to height $h$. Can give pressure reading in “mm of Hg”
End of Lecture 25

- For Friday, Nov. 6, read Walker 15.3-5.
- Homework Assignment 13b is due at 11:00 PM on Sunday, Nov. 8.