Physics 111
Lecture 23 (Walker:11.3-7)
Equilibrium of Solid Objects
Balance
Angular Momentum
Conservation of Angular Momentum

Oct. 30, 2009

Quiz Wed. - Chaps. 9,10,11

Equilibrium Example: Ladder
10 m ladder weighing 50N placed against smooth wall at 50° angle. Find forces from floor & wall, and the μ needed for equilibrium?

Torque Condition:
\[ \sum \tau = mg \left( \frac{L}{2} \right) \cos 50° + PL \sin 50° = 0 \]
\[ P \sin 50° = (25 N) \cos 50° \]
\[ P = 21N \]

Force Conditions:
\[ \sum F_x = 0 = f - P \]
\[ f = P = 21N \]
\[ \sum F_y = 0 = n - mg \]
\[ n = mg = 50N \]
\[ f \leq \mu n \, \text{so need } \mu \geq (f/n) \text{ or } \mu \geq 0.42 \]

Solving Statics Problems
If there is a cable or cord (or muscle) in the problem, it can support forces only along its length. Forces perpendicular to that would cause it to bend.
Applications to Muscles and Joints

These same principles can be used to understand forces within the body.

Force to Hold Baseball (Prob. 11-5)

Person holds 1.4N baseball in hand, 0.34 m from elbow joint. Biceps, attached at 2.75 cm from elbow, exerts 12.6 N force. Take forearm & hand to be uniform rod of mass 1.2 kg. Find net torque on forearm & hand.

Take axis to be elbow joint.

\[ \Sigma \tau = (0.0275\text{m})(12.6\text{N}) - (0.17\text{m})(1.2\text{kg})(9.8\text{N/kg}) - (0.34\text{m})(1.4\text{N}) = -2.1 \text{ Nm} \]

Applications to Muscles and Joints

The angle at which this man's back is bent places an enormous force on disks at base of his spine, as the lever arm for \( F_M \) is so small.

Center of Mass & Balance

If an extended object is to be balanced, it must be supported through its center of mass.
Balance

An object balances when the pivot point is located directly under (or over) the center of mass. For that situation, $\tau_{\text{grav}} = 0$.

Center of Mass & Balance

This fact can be used to find the center of mass of an object - suspend it from different axes and trace a vertical line. The center of mass is where the lines meet.

Angular Momentum

Definition of the Angular Momentum, $L$

$L = I \omega$

SI unit: kg m$^2$/s

$p = mv$ \hspace{1cm} $L = I \omega$

For particle of mass $m$ moving in a circle of radius $r$, $I = mr^2$ and $\omega = v/r$, so

$L = rmv = rp$

Angular Momentum - Point Particle

For more general motion of a point particle,

$L = r \perp p = r \perp mv$
**Examples: Angular Momentum**

(a) What is angular momentum of 0.13 kg Frisbee, considered to be uniform disk of radius 7.5 cm, spinning at $\omega = 11.5 \text{ rad/s}$?

$$I_{\text{frisbee}} = \frac{1}{2}mr^2 = \frac{1}{2}(0.13 \text{ kg})(0.075 \text{ m})^2 = 3.66 \times 10^{-4} \text{ kg m}^2$$

$$L_{\text{frisbee}} = I_{\text{frisbee}}\omega = (3.66 \times 10^{-4} \text{ kg m}^2)(11.5 \text{ rad/s}) = 4.2 \times 10^{-3} \text{ kg m}^2/\text{s}$$

(b) What is angular momentum of 95 kg person running with speed of 5.1 m/s around circular track of radius 25 m?

$$L_{\text{runner}} = rmv = (25.0 \text{ m})(95 \text{ kg})(5.1 \text{ m/s}) = 12,000 \text{ kg m}^2/\text{s}$$

**Question**

An object is moving in a straight line with momentum $p$. It has non-zero angular momentum:

(a) always;     (b) sometimes;     (c) never.

The value of $L$ depends on where we take the axis of rotation.

An object is moving in a straight line with momentum $p$ has non-zero angular momentum:

(a) always;     (b) sometimes;     (c) never.

**Sign of Angular Momentum**

The angular momentum $L$ is take to be positive if the angular position is increasing with time, i.e., if the motion associated with $L$ is a counterclockwise rotation.
Spin Angular Momentum of Earth

What is the angular momentum of Earth as it rotates on its axis?

\[
\omega = \frac{1 \text{ rev}}{24 \text{ hr}} \times \frac{1 \text{ hr}}{3600 \text{ s}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} = 7.27 \times 10^{-5} \text{ rad/s}
\]

\[
I = \frac{2}{5} M_e R_e^2 = \frac{2}{5} (5.98 \times 10^{24} \text{ kg})(6.37 \times 10^6 \text{ m})^2 = 9.71 \times 10^{37} \text{ kg m}^2
\]

\[
L = I\omega = 7.06 \times 10^{33} \text{ kg m}^2/\text{s}
\]

Orbital Angular Momentum of Earth

What is the angular momentum of Earth as it orbits the Sun?

\[
\omega_{\text{orb}} = \frac{1 \text{ orbit}}{1 \text{ yr}} \times \frac{1 \text{ yr}}{365.25 \text{ dy}} \times \frac{1 \text{ dy}}{24 \times 60 \times 60 \text{ s}} \times \frac{2\pi \text{ rad}}{1 \text{ orbit}} = 1.991 \times 10^{-7} \text{ rad/s}
\]

\[
I_{\text{orb}} = M_e r_{\text{orb}}^2 = (5.98 \times 10^{24} \text{ kg})(1.50 \times 10^{11} \text{ m})^2 = 1.346 \times 10^{47} \text{ kg m}^2
\]

\[
L_{\text{orbit}} = I_{\text{orb}} \omega_{\text{orbit}} = 2.68 \times 10^{48} \text{ kg m}^2/\text{s}
\]

Changing Angular Momentum

Looking at the rate at which angular momentum changes,

\[
\frac{\Delta L}{\Delta t} = I \frac{\Delta \omega}{\Delta t} = I \alpha
\]

Newton’s Second Law for Rotational Motion

\[
\tau = I \alpha = \frac{\Delta L}{\Delta t}
\]

Therefore, if \( \tau = 0 \), then \( L \) is constant with time.

Example: A Windmill

In light wind, windmill experiences constant torque of 255 N m. If windmill is initially at rest, what is its angular momentum after 2.00 s?

\[
\tau = \frac{\Delta L}{\Delta t}
\]

\[
\Delta L = \tau \Delta t = (255 \text{ N})(2.00 \text{ s}) = 510 \text{ kg m}^2/\text{s}
\]

Notice that you do not need to know the moment of inertia of the windmill to do this calculation.
Conservation of Angular Momentum

If the net external torque on a system is zero, the angular momentum is conserved.

\[ L_i = L_f \implies I_i \omega_i = I_f \omega_f \]

Example: Spinning the Wheel

You are sitting on a stool on frictionless turntable holding a bicycle wheel. Initially, neither the wheel nor the turntable is spinning. You hold the axle vertical with one hand and spin the wheel counterclockwise with the other hand.

You observe that the stool and turntable begin to rotate clockwise. Then you stop the wheel with your free hand. What happens to the turntable rotation?

\[ L_1 = L_2 = L_3 \implies \text{The turntable stops.} \]

Example: A Stellar Performance

Star of radius \( R_i = 2.3 \times 10^8 \text{ m} \) rotates initially with an angular speed of \( \omega_i = 2.4 \times 10^{-6} \text{ rad/s} \).

If the star collapses to a neutron star of radius \( R_f = 20.0 \text{ km} \), what will be its final angular speed \( \omega_f \)?

\[ L_i = L_f \implies I_i \omega_i = I_f \omega_f \]

\[ \omega_f = \left( \frac{I_i}{I_f} \right) \omega_i = \frac{\frac{2}{3} MR_i^2}{\frac{2}{3} MR_f^2} \omega_i = \left( \frac{R_i^2}{R_f^2} \right) \omega_i \]

\[ = \left[ \frac{(2.3 \times 10^8 \text{ m})^2}{(2.0 \times 10^4 \text{ m})^2} \right] (2.4 \times 10^{-6} \text{ rad/s}) = 320 \text{ rad/s} \]

\[ = 3056 \text{ rpm} \]
Oscillations have some common characteristics:
1. They take place around an equilibrium position;
2. The motion is periodic and repeats with each cycle.

Periodic Motion Characteristics

| Period \( T \): time required for one cycle of periodic motion (unit: s) |
| Frequency \( f \): number of cycles per unit time |

\[
f = \frac{1}{T} \quad \text{SI unit: cycle/second} = 1/s = s^{-1}
\]

The frequency unit is called a hertz (Hz): 1 Hz = 1 cycle/second

| Amplitude \( A \): maximum distance object moves from equilibrium (unit: m) |

Frequency and Period

\[
f = \frac{1}{T} \quad \text{and} \quad T = \frac{1}{f}
\]

\( T \) is the period (units: s) \( f \) is the frequency
(units: Hz \( \equiv \) oscillations per second)

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<thead>
<tr>
<th>TABLE Units of frequency</th>
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<tbody>
<tr>
<td>Frequency</td>
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<tr>
<td>(10^3 \text{ Hz} = 1 \text{ kilohertz} = 1 \text{ kHz} )</td>
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<tr>
<td>(10^6 \text{ Hz} = 1 \text{ megahertz} = 1 \text{ MHz} )</td>
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<td>(10^9 \text{ Hz} = 1 \text{ gigahertz} = 1 \text{ GHz} )</td>
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