Conservation of (System) Momentum

When no external forces do work on a system consisting of objects that interact with each other, the total momentum of the system before the interaction is equal to the total momentum of the system after the interaction.

Example: A Space Repair
Astronaut pushes solar panel, giving it velocity of -0.30 m/s in x-direction. Astronaut's mass is 60 kg; panel's mass 80 kg. Both astronaut and panel initially at rest. What is astronaut's velocity after push?

External forces do no work, so system momentum conserved.

\[ m_P \vec{v}_{pf} + m_A \vec{v}_{Af} = m_P \vec{v}_{Pi} + m_A \vec{v}_{Ai} = 0 \]
\[ m_P \vec{v}_{pf} = -m_A \vec{v}_{Af} \]
\[ \vec{v}_{Af} = -\frac{m_P}{m_A} \vec{v}_{pf} = -\frac{(80 \text{ kg})}{(60 \text{ kg})}(-0.30 \text{ m/s}) = (0.40 \text{ m/s}) \hat{x} \]
Example: Runaway Railroad Car

Runaway 10,000 kg railroad car is rolling horizontally at 4.00 m/s toward a switchyard. As it passes grain elevator, 2,000 kg of grain suddenly drops into car. Assume that grain drops vertically and that rolling friction and air drag are negligible. How fast is car going after grain drops in?

\[
\begin{align*}
\nonumber
m_c &= 10,000 \text{ kg} \\
m_g &= 2,000 \text{ kg} \\
v_{ix} &= 4.00 \text{ m/s} \\
mg &= 2,000 \text{ kg} \\

\end{align*}
\]

\[
\begin{align*}
\nonumber
\text{After:} \\

m_c + m_g \\

\nonumber
v_{fx} = \frac{(m_c + m_g)v_{ix} - m_g v_{ix} + m_g (0)}{m_c + m_g} \\

v_{fx} = \frac{v_{ix} m_c}{m_c + m_g} \\

v_{fx} = \frac{(4.0 \text{ m/s})(10,000 \text{ kg})}{(12,000 \text{ kg})} \\

v_{fx} = 3.3 \text{ m/s}
\end{align*}
\]

Example: Runaway Railroad Car

External forces do no work, so momentum of system (car & grain) is conserved.

\[
\begin{align*}
\nonumber
P_{fx} = P_{ix} \\

(m_c + m_g)v_{fx} = m_c v_{ix} + m_g (0) \\

v_{fx} = v_{ix} \frac{m_c}{m_c + m_g} \\

v_{fx} = (4.0 \text{ m/s})(10,000 \text{ kg})/(12,000 \text{ kg}) \\

v_{fx} = 3.3 \text{ m/s}
\end{align*}
\]

Collisions

Collision: two objects striking one another. Time of collision is short enough that external forces may be ignored; system momentum conserved.

What about system mechanical energy?

Elastic collision: momentum and kinetic energy conserved; \( p_f = p_i \) & \( K_f = K_i \)

Inelastic collision: momentum is conserved but kinetic energy is not: \( p_f = p_i \) but \( K_f \neq K_i \)

Completely inelastic collision: objects stick together afterwards: \( p_f = p_{i1} + p_{i2} \)

Sketches for Collision Problems

- Draw "before" and "after" sketches
- Label each object
  - include the direction of velocity
  - keep track of subscripts
Sketch: Perfectly Inelastic Collisions

- The objects stick together
- Include all the velocity directions
- The “after” collision combines the masses

Inelastic Collisions

A completely inelastic collision:

\[ m_1 v_{1,i} + m_2 v_{2,i} = (m_1 + m_2) v_f \]

Example: Goal-Line Stand

95.0 kg running back runs forward at 3.75 m/s, 111 kg line backer moving at 4.10 m/s meets runner in head-on collision and locks arms around runner.
(a) Find their velocity immediately after collision.
(b) Find initial & final kinetic energies.

\[ v_f = \frac{m_1 v_{1,i} + m_2 v_{2,i}}{m_1 + m_2} \]

\[ K_i = \frac{1}{2} m_1 v_{1,i}^2 + \frac{1}{2} m_2 v_{2,i}^2 = 1600 \text{J} \]

\[ K_f = \frac{1}{2} (m_1 + m_2) v_f^2 = 24 \text{J} \]
Explosions

An explosion in which particles of system move apart from each other after brief, intense interaction, is the opposite of a collision. The explosive forces are internal forces. If the system is isolated, its total momentum will be conserved during explosion, so net momentum of fragments equals initial momentum.

Example: Recoil of Rifle

10 g bullet fired from 3.0 kg rifle with speed of 500 m/s. What is recoil speed of rifle?

\[ P_x = 0 \]

\[ P_{fx} = m_B(v_{fx})_{bullet} + m_R(v_{fx})_{rifle} = P_{ix} = 0 \]

\[ (v_{fx})_{rifle} = -\frac{m_B}{m_R} (v_{fx})_{bullet} \]

\[ = -\frac{(0.010 \text{ kg})}{(3.0 \text{ kg})} (500 \text{ m/s}) = -1.67 \text{ m/s} \]

Elastic Collisions

In elastic collisions, both kinetic energy and momentum are conserved.

One-dimensional elastic collision:

Elastic Collisions in 1D*

\[ \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 = \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 \]

Momentum Conservation

\[ m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i} + m_2 v_{2i} \]
Elastic Collisions*

We have two equations (conservation of momentum and conservation of kinetic energy) and two unknowns (the final speeds). Solving for the final speeds:

\[ v_{1,f} = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) v_0 \]
\[ v_{2,f} = \left( \frac{2m_1}{m_1 + m_2} \right) v_0 \]

Elastic Collisions: 3 Cases*

\[ (v_{fx})_2 = \frac{2m_1}{m_1 + m_2} (v_{fx})_1 \]
\[ (v_{fy})_2 = \frac{m_1 - m_2}{m_1 + m_2} (v_{fy})_1 \]

\[ m_1 = m_2: \quad (v_{fx})_2 = (v_{fx})_1 \text{ and } (v_{fy})_1 = 0 \text{ (knock-on)} \]

Ball 1 stops. Ball 2 goes forward with \( v_2 = v_1 \).

Center of Mass

The center of mass of a system is the point where the system can be balanced in a uniform gravitational field.
**Center of Mass**
For two objects:

\[ X_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{m_1 x_1 + m_2 x_2}{M} \]

The center of mass is closer to the more massive object.

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**Center of Mass**
The center of mass need not be within the object:

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**Center of Mass Motion**
Total mass multiplied by the acceleration of the center of mass is equal to the net external force:

**Newton's Second Law for a System of Particles**

\[ M \ddot{X}_{cm} = \vec{F}_{net,ext} \]

Center of mass accelerates as though it were a point particle of mass \( M \) acted on by \( \vec{F}_{net,ext} \)

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**End of Lecture 19**
- Homework Assignment #9b is due at 11:00 PM on Tuesday, Oct. 20.