Physics 111
Lecture 18 (Walker: 9.1-3)
Momentum & Impulse
Conservation of Momentum
Oct. 14, 2009

Friday - Quiz on Chaps. 7 and 8

Momentum
• From Newton's laws: force must be present to change an object's velocity (speed and/or direction)
  ➢ Wish to consider changes in velocity due to collisions, when force varies in time in a complicated way
• Best method is to use concept of linear momentum

Linear Momentum
Definition of Linear Momentum, \( \vec{p} \)

\[
\vec{p} = m\vec{v}
\]

SI unit: kg \( \cdot \) m/s
Momentum is a vector; its direction is the same as the direction of the velocity.
• Can write in component form; for two-dimensional motion:

\[ p_x = mv_x \text{ and } p_y = mv_y \]

Momentum Example
• A 0.25 kg baseball is moving in the x-direction at 10 m/s. What is its momentum?
• \( \vec{p} = m\vec{v} = (0.25 \text{ kg})(10 \text{ m/s } \hat{x}) \)
  \[ = 2.5 \text{ kg}\cdot \text{m/s } \hat{x} \]
  or \( p_x = 2.5 \text{ kg}\cdot \text{m/s } ; \) \( p_y = 0 \)
Change in Momentum

Change in momentum: \( \Delta p_y = p_{yf} - p_{yi} \)

Teddy Bear: \( \Delta p_y = 0 - (-mv) = mv \)

Bouncing Ball: \( \Delta p_y = mv - (-mv) = 2mv \)

Momentum & Newton's Second Law*

Newton's second law, as we wrote it before:

\[
\sum \vec{F} = m \vec{a} = m \frac{\Delta \vec{v}}{\Delta t} = \frac{\Delta (mv)}{\Delta t} = \frac{\Delta \vec{p}}{\Delta t}
\]

is only valid for objects that have constant mass.

Here is a more general form in terms of momentum, also useful when the mass is changing:

**Newton's Second Law**

\[
\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t}
\]

Impulse

- In order to change the momentum of an object (say, baseball), a force must be applied.
- The time rate of change of momentum of an object is equal to the net force acting on it:
  \[
  \vec{F}_{net} = \frac{\Delta \vec{p}}{\Delta t} \quad \text{or} \quad \Delta \vec{p} = \vec{F}_{net} \Delta t
  \]
  - Gives an alternative statement of Newton's second law.
  - \((F \Delta t)\) is defined as the impulse.

Definition of Impulse, \( \vec{I} \)

\[
\vec{I} = \vec{F}_{av} \Delta t
\]

SI unit: N \cdot s = kg \cdot m/s

Impulse is a vector, in the same direction as the average force.
Impulse

We can rewrite \( \overrightarrow{F}_{av} = \frac{\Delta \overrightarrow{p}}{\Delta t} \)
as
\( \overrightarrow{F}_{av} \Delta t = \Delta \overrightarrow{p} \)
So we see that \( \vec{I} = \overrightarrow{F}_{av} \Delta t = \Delta \overrightarrow{p} \)

The impulse \( \vec{I} \) exerted on an object is equal to the change in object's momentum \( \Delta \overrightarrow{p} \).

Question 1

A 10 kg cart collides with a wall and changes its direction. What is its change in x-momentum \( \Delta p_x \)?

a. \(-30 \text{ kg m/s}\)
b. \(-10 \text{ kg m/s}\)
c. \(10 \text{ kg m/s}\)
d. \(20 \text{ kg m/s}\)
e. \(30 \text{ kg m/s}\)

What was the impulse exerted on the cart by the wall??

Graphical Interpretation of Impulse

• Usually force not constant, but time-dependent

\[ \text{Impulse equals area under curve} \]

• If force not constant, use average force \( \overrightarrow{F} = \overrightarrow{F}_{avg} \)
• Average force can be thought of as constant force that would give same impulse to object in the time interval as actual time-varying force.

Impulse and Average Force

Definition of Impulse: \( \vec{I} \equiv \overrightarrow{F}_{av} \Delta t \)

\[ \vec{I}_{net} = \Delta \overrightarrow{p} \]

\[ \vec{I}_{ext} = \overrightarrow{F}_{av ext} \Delta t = \Delta \overrightarrow{p}_{sys} \]

\[ \vec{I} = \overrightarrow{F}_{av} \Delta t = \text{area under F vs. t curve} \]
Problem Solving Strategy

**Picture:** To estimate average force \( F_{av} \), first estimate impulse \( I \) of force. Impulse is equal to change in object’s momentum, i.e., mass times change in velocity. (An estimate of velocity change \( \Delta v \) can be made from estimates of collision time \( \Delta t \) and displacement \( \Delta r \).)

**Solve:**
1. Draw sketch showing before and after positions of object.
2. Calculate impulse from momentum change during collision. \( (I = \Delta p = m \Delta v) \)
3. Use \( F_{av} = I / \Delta t \) to calculate average force.

**Check:** Average force is a vector, and should be in same direction as \( \Delta v \).

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Impulse

The same change in momentum may be produced by a large force acting for a short time, or by a smaller force acting for a longer time.

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Problem: Teeing Off

A 50-g golf ball at rest is hit by “Big Bertha” club with 500-g mass. After the collision, ball leaves with velocity of 50 m/s.

a) Find impulse imparted to ball
b) Assuming club in contact with ball for 0.5 ms, find average force acting on golf ball

**Given:**
- mass: \( m = 50 \text{ g} = 0.050 \text{ kg} \)
- velocity: \( v = 50 \text{ m/s} \)

**Find:**
- impulse = ?
- \( F_{av} = ? \)

1. Use impulse-momentum relation:

   \[
   \text{impulse} = \Delta p = mv_f - mv_i
   \]

   \[
   = (0.050 \text{ kg})(50 \text{ m/s}) - 0
   \]

   \[
   = 2.50 \text{ kg} \cdot \text{m/s}
   \]

2. Having found impulse, find the average force from the definition of impulse:

   \[
   \Delta p = F \cdot \Delta t, \text{ thus } F = \frac{\Delta p}{\Delta t} = \frac{2.50 \text{ kg} \cdot \text{m/s}}{0.5 \times 10^{-3} \text{ s}}
   \]

   \[
   = 5.00 \times 10^3 \text{ N}
   \]

**Note:** according to Newton’s 3rd law, there is also a reaction force on club.
Example: Hitting a Baseball (1)

150 g baseball is thrown at speed 20 m/s. It is hit straight back to pitcher at speed 40 m/s. The interaction force is as shown here.
What is average force \( F_{av} \) that bat exerts on ball? (Neglect all other forces on ball during the brief duration of collision.)

What is maximum force \( F_{max} \) that bat exerts on ball?

\[
F_t(t) = F_{max}
\]

\[
F_{av} = \frac{\Delta p_x}{\Delta t} = \frac{(9.0 \text{ kg m/s})}{0.003 \text{ s}} = 3,000 \text{ N}
\]

Example: Karate Blow

With a karate blow, you shatter concrete block. Your hand has mass 0.70 kg, is initially moving downward at 5.0 m/s, and stops 6.0 mm beyond point of contact.
(a) What impulse does block exert on hand?
(b) What is approximate collision time and average force that block exerts on hand?

\[
\Delta v = 5.0 \text{ m/s} \hat{y} \quad \Delta v = v_{av}\Delta t \approx \frac{1}{2} \Delta v\Delta t
\]

\[
\vec{I} = \Delta \vec{p} = m\Delta \vec{v} = (0.70 \text{ kg})(5.0 \text{ m/s} \hat{y}) = 3.5 \text{ N s} \hat{y}
\]

\[
\Delta t = \Delta v = \frac{2\Delta v}{\Delta v} = \frac{2(0.006 \text{ m})}{5.0 \text{ m/s}} = 0.00240 \text{ s}
\]

\[
\vec{F}_{av} = \frac{\vec{I}}{\Delta t} = \frac{(3.5 \text{ N s} \hat{y})}{(0.00240 \text{ s})} = 1,500 \text{ N} \hat{y}
\]

Impulse Applied to Auto Collisions

- The most important factor is the collision time or the time it takes the person to come to rest
  - Increased collision time reduces injury in car crash
- Ways to increase the time
  - Crumple zone at front of car
  - Air bags

\[
\vec{F}_{av} \text{ and } F_{max}
\]
Example: A Crumpled Car
Car with 80 kg crash dummy drives into concrete wall at 25 m/s (56 mi/h).
(a) Estimate displacement of dummy during crash.
(b) Estimate average force that seat belt exerts on dummy.
\[
\vec{F}_{av} \Delta t = \Delta \vec{p} = m \Delta \vec{v}
\]
\[
\Delta t = \Delta x / v_{av} = \Delta x / \frac{1}{2} \Delta v = (1.0 \text{ m}) / \frac{1}{2} (25 \text{ m/s}) = 0.080 \text{ s}
\]
\[
\vec{F}_{av} = I / \Delta t = (-2000 \text{ N s } \hat{x}) / (0.080 \text{ s}) = -25,000 \text{ N } \hat{x}
\]

Conservation of Linear Momentum
The net force acting on an object is the rate of change of its momentum:
\[
\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t}
\]
If net force is zero, momentum does not change:

Internal versus External Forces:
Internal forces act between objects within the system. As with all forces, they occur in action-reaction pairs. As all pairs act between objects in the system, the internal forces always sum to zero:
\[
\sum \vec{F}_{\text{int}} = 0
\]
Therefore, the net force acting on a system is the sum of the external forces acting on it.
**Conservation of (System) Momentum**

- Mathematically: $m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$
  - Momentum is conserved for an isolated system of objects
  - The system includes all the objects interacting with each other
  - Assumes only internal forces are acting during the collision
  - Can be generalized to any number of objects

**Conservation of Linear Momentum**

An example of internal forces moving components of a system:

**ConcepTest**

Suppose a person jumps on the surface of Earth. The Earth

1. Will not move at all
2. Will recoil in the opposite direction with tiny velocity
3. Might recoil, but there is not enough information provided to see if that could happened

**End of Lecture 18**

- Before Friday, read *Walker* 9.4-7. (Skip material on two-dimensional collisions and on Motion of Center of Mass.)
- Homework Assignments #9a should be submitted using WebAssign by 11:00 PM on Sunday, Oct. 18.
- Friday - Quiz on Chaps. 7 and 8