Physics 111

Lecture 17 (Walker: 8.3-4)

Conservation of Mechanical Energy

Oct. 12, 2009

Friday - Quiz on Chaps. 7 and 8

Conservation of Mechanical Energy

Definition of mechanical energy $E$:

$$ E = U + K $$

(8-6)

If the only work done in going from the initial to the final position is done by conservative forces:

$$ E_f = E_i $$

When only conservative forces act

Or equivalently:

$$ E = \text{constant} $$

Example Problem

Tarzan swings on a 30.0-m-long vine initially inclined at angle 35.0° with vertical. What is his speed at bottom of swing (a) if he starts from rest? (b) if he pushes off with speed 4.00 m/s?
Choose zero of potential energy at bottom of arc. Tarzan’s initial height above this level is:

\( y_i = 30m[1 - \cos(35^\circ)] = 5.4 \text{ m} \)

Two forces act during the swing, gravity (conservative) and vine tension (not conservative, but does no work). Thus \( E_f = E_i \).

a) \( \frac{1}{2}mv_f^2 + 0 = \frac{1}{2}mv_i^2 + mgy_i = 0 + mgy_i \)

\( v_f^2 = 2gy_i = 2(9.8\text{m/s}^2)(5.4\text{m}) \) & thus \( v_f = 10.3 \text{ m/s} \)

b) \( \frac{1}{2}mv_f^2 + 0 = \frac{1}{2}mv_i^2 + mgy_i \)

\( v_f^2 = v_i^2 + 2gy_i = (4.0\text{m/s})^2 + 2(9.8\text{m/s}^2)(5.4\text{m}) \); \( v_f = 11.0 \text{ m/s} \)

**Using Conservation of Mechanical Energy to Solve Problems**

- **MODEL**: Choose a system without friction losses or other losses of mechanical energy.
- **VISUALIZE**: Draw a before-and-after pictorial representation.
- **SOLVE**: The mathematical representation is based on the law of conservation of mechanical energy:
  \( E_f = E_i \)
- **ASSESS**: Check that your result has the correct units, is reasonable, and answers the question.

**Question**

When a ball of mass \( m \) is dropped from height \( h \), its kinetic energy just before landing is \( K \).

If a 2nd ball of mass \( 2m \) is dropped from height \( h/2 \), what is its kinetic energy just before landing?

\[ (a) \ K/4 \quad (b) \ K/2 \quad (c) \ K \quad (d) \ 2K \quad (e) \ 4K \]

**What is the Final Speed?**

v = 0

v = 4 m/s

v = 1 m/s, 2 m/s, or 3 m/s?

v = 5 m/s
Energy is a scalar. Speed of cap is $v_i$ at height $y_i$ and $v_f$ at height $y_f$, independent of the path between the two heights. Angle at which cap is launched does not matter, as long as $v_i$ is large enough to carry cap to height $y_f$.

Example: Catching a Home Run

Player hits 0.15 kg baseball over outfield fence. Ball leaves bat with speed of 36.0 m/s and a fan in bleachers catches it 7.2 m above the point where it was hit. Neglect air resistance.
(a) What is the kinetic energy $K_f$ of the ball when caught?
(b) What is the speed $v_f$ of the ball when caught.

\[ E = U_i + K_i = mgy_i + \frac{1}{2}mv_i^2 \]
\[ = 0 + \frac{1}{2}(0.15\text{ kg})(36\text{ m/s})^2 = 97\text{ J} \]
\[ U_f = mgy_f = (0.15\text{ kg})(9.81\text{ m/s}^2)(7.2\text{ m}) = 11\text{ J} \]
\[ K_f = E - U_f = (97\text{ J}) - (11\text{ J}) = 86\text{ J} \]
\[ v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(86\text{ J})}{(0.15\text{ kg})}} = 34\text{ m/s} \]
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- $E_t = E_h$
- $\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_h^2 + mgy_h$
- $\frac{1}{2}m(rg) + mg(2r) = 0 + mgh$
- $h = 2r + \frac{1}{2}r = 2.5r$

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**Example: Spring Time**

Block of mass $m$ slides on horizontal, frictionless surface until it hits a spring with force constant $k$. Block comes to rest after compressing spring a distance $d$. Find initial speed of block. (Ignore air resistance and energy lost when block initially hits spring.)

![Diagram of block sliding on a horizontal surface, compressing a spring a distance $d$.]

- Only the spring force does work, and it is conservative.
- Thus $E_f = E_i$
  
  \[ \frac{1}{2}kx_f^2 + \frac{1}{2}mv_f^2 = \frac{1}{2}kx_i^2 + \frac{1}{2}mv_i^2 \]
  
  \[ \frac{1}{2}kd^2 + 0 = 0 + \frac{1}{2}mv_i^2 \]
  
  \[ v_i^2 = \frac{kd^2}{m} \]
  
  \[ v_i = d \sqrt{\frac{k}{m}} \]

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Basic Energy Model

1. Kinetic energy $K$ is associated with motion of a particle. Potential energy $U$ is associated with position of particle when conservative force acts.

2. Kinetic energy can be transformed into potential energy, and potential energy into kinetic energy.

3. Under *some* circumstances, mechanical energy $E = K + U$ is conserved -- its value at end of a process is same as at beginning.

Q1: Under what circumstances is $E$ conserved?
Q2: What happens to the energy when $E$ is not conserved?

Nonconservative Forces

In the presence of *nonconservative* forces, the total mechanical energy is *not* conserved:

$$W_{\text{total}} = W_{c} + W_{nc}$$

$$= -\Delta U + W_{nc} = \Delta K$$

Solving,

$$W_{nc} = \Delta U + \Delta K = \Delta E$$ (8-9)

Example: Judging a Putt

Golfer badly misjudges putt, giving ball initial speed $v_1$ which sends ball distance $d$ that is 1/4 distance to hole.

If nonconservative force $F$ due to resistance of grass is constant, what initial speed $v_2$ is needed to putt ball from initial position to hole?

$$\Delta E_1 = K_f - K_i = 0 - \frac{1}{2}mv_1^2 = -Fd$$

$$\Delta E_2 = -\frac{1}{2}mv_2^2 = -F(4d)$$

∴ $v_2 = 2v_1$

Example: Landing with a Thud

Block of mass $m_1 = 2.40$ kg is on horizontal table with coefficient of friction $\mu_k = 0.450$ and is connected to hanging block of mass $m_2 = 1.80$ kg. When blocks are released, they move distance $d = 0.50$ m and then $m_2$ hits the floor.

Find speed of blocks just before $m_2$ hits.
$U_i = m_1gh + m_2gd; \quad K_i = 0; \quad E_i = m_1gh + m_2gd$
$U_f = m_1gh + m_2g(0); \quad K_f = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2; \quad E_f = m_1gh + \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$
$\Delta E = E_f - E_i = \frac{1}{2} (m_1 + m_2)v_1^2 - m_2gd$
$W_{nc} = -f_kd = -\mu_km_1gd$
$\Delta E = W_{nc} \Rightarrow \frac{1}{2}(m_1 + m_2)v_1^2 - m_2gd = -\mu_km_1gd$
$v = \sqrt{\frac{2gd(m_2 - \mu_km_1)}{m_1 + m_2}} = \sqrt{\frac{2(9.81 \text{ m/s}^2)(0.50 \text{ m})[(1.80 \text{ kg}) - (0.45)(2.40 \text{ kg})]}{(2.40 \text{ kg}) + (1.80 \text{ kg})}}$
$= 1.30 \text{ m/s}$

**Momentum**

- From Newton's laws: force must be present to change an object's velocity (speed and/or direction)
- Wish to consider changes in velocity due to collisions, when force varies in time in a complicated way
- Best method is to use concept of linear momentum

Linear momentum = product of mass $\times$ velocity