Work Done by a Constant Force

If there is more than one force acting on an object, we can add the work done by each force, or just find the work done by the net force:

\[ W_{\text{total}} = (F_{\text{total}} \cos \theta)d = F_{\text{total}}d \cos \theta \]

(7-5)

Example: Pulling a Suitcase

Rope inclined upward at 45° pulls suitcase through airport. Tension on rope is 20 N.

How much work does the tension do if suitcase is pulled 100 m?

\[ W = T(\Delta x) \cos \theta \]

\[ W = (20 \text{ N})(100 \text{ m}) \cos 45^\circ = 1410 \text{ J} \]

Note that the same work could have been done by a tension of just 14.1 N by pulling in the horizontal direction.

Gravitational Work

In lifting object of weight \( mg \) by a height \( h \) at constant slow speed, person doing lifting does work \( W = mgh \).

If the object is subsequently allowed to fall a distance \( h \), gravity does work \( W = mgh \) on the object.
**Example: Loading Ship**

3,000 kg truck is to be loaded onto a ship by crane that exerts upward force of 31 kN on truck. This force is applied over a distance of 2.0 m.

(a) Find work done on truck by crane.

(b) Find work done on truck by gravity.

(c) Find net work done on the truck.

\[
W_{\text{app}} = F_{\text{app}} \Delta y = (31 \text{ kN})(2.0 \text{ m}) = 62 \text{ kJ}
\]

\[
W_{g} = mg \Delta y = (3000 \text{ kg})(-9.81 \text{ m/s}^2)(2.0 \text{ m}) = -58.9 \text{ kJ}
\]

\[
W_{\text{net}} = W_{\text{app}} + W_{g} = (62.0 \text{ kJ}) + (-58.9 \text{ kJ}) = 3.1 \text{ kJ}
\]

**Kinetic Energy**

Algebraic manipulations of the equations of motion relate the net work done on an object of mass \( m \) to its change of speed:

\[
W_{\text{total}} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2
\]

Therefore, we define the kinetic energy \( K \):

\[
K = \frac{1}{2} m v^2
\]

**Positive & Negative Gravity Work**

When positive net work done on an object, its speed increases; when negative work is done, its speed decreases.

**Question 1**

Car 1 has twice the mass of Car 2, but they both have the **same** kinetic energy. If the speed of Car 2 is \( v \), approximately what is the speed of Car 1?

a) 0.50 \( v \)  
 b) 0.707 \( v \)  
 c) \( v \)  
 d) 1.414 \( v \)  
 e) 2.00 \( v \)
Energy

- Forms of energy:
  - Mechanical
    - Kinetic, Potential: focus for now
  - Thermal
  - Chemical
  - Electromagnetic
  - Nuclear
- Energy can be transformed from one form to another
  - Essential to the study of physics, chemistry, biology, geology, astronomy
- Can be used in place of Newton’s laws to solve certain problems more simply
- Energy units: SI Unit - Joule (J); Calorie (food calorie) = 4.2 kJ; Kilowatt-hour = 3.6 MJ

Kinetic Energy & The Work-Energy Theorem

Work-Energy Theorem: The total work done on an object is equal to its change in kinetic energy.

\[ W_{\text{total}} = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \]  

Problem Solving Strategy - Work

1. Draw particle (as dot) at its initial and final positions.
2. Label initial and final positions.
3. Put one or more coordinate axes on the drawing.
4. Draw arrows for initial and final velocities, and label them.
5. On the initial-position drawing, place a labeled vector for each force acting on object.
6. Calculate the total work done on the particle by the forces and equate this total to the change in the particle’s kinetic energy.

Check: Pay attention to negative signs. Values for work done can be positive or negative, depending on the direction of the displacement relative to the direction of the force. Kinetic energy values are always positive.
**Example: A Dogsled Race**

You pull a sled (mass 80 kg) with force of 180 N at 40° above the horizontal. Sled moves \( \Delta x = 5.0 \) m, starting from rest. Neglect friction. (a) Find the work you do. (b) Find final speed of sled.

\[
W_{\text{total}} = W_{\text{you}} = F \Delta x = F \cos \theta \Delta x = (180 \text{ N})(\cos 40^\circ)(5.0 \text{ m}) = 689 \text{ J}
\]

\[
\begin{align*}
v_f^2 &= \frac{2W_{\text{total}}}{m} = \frac{2(689 \text{ J})}{80 \text{ kg}} = 4.15 \text{ m/s} \\
v_f &= \sqrt{\frac{2W_{\text{total}}}{m}} = \sqrt{\frac{2(689 \text{ J})}{80 \text{ kg}}} = 4.15 \text{ m/s}
\end{align*}
\]

---

**Example: Work & KE-Rocket Launch**

150,000 kg rocket launched straight up. Rocket engine generates thrust of \( 4.0 \times 10^6 \) N. Rocket's speed at height 500 m? (Ignore air resistance and mass loss due to burned fuel.)

\[
W_{\text{thrust}} = F_{\text{thrust}}(\Delta y) = (4.0 \times 10^6 \text{ N})(500 \text{ m}) = 2.0 \times 10^9 \text{ J}
\]

\[
W_{\text{grav}} = -\Delta y = -mg(\Delta y) = -(1.5 \times 10^5 \text{ kg})(9.8 \text{ N/kg})(500 \text{ m}) = -0.74 \times 10^9 \text{ J}
\]

\[
\Delta K = \frac{1}{2}mv_i^2 - 0 = W_{\text{thrust}} + W_{\text{grav}} = 1.26 \times 10^9 \text{ J}
\]

\[
v = \sqrt{\frac{2\Delta K}{m}} = 130 \text{ m/s}
\]
Graphical Interpretation of Work

If force is constant, we can interpret the work done by it in moving from \( x_1 \) to \( x_2 \) graphically:

\[
\text{Area} = Fd = W
\]

Work Done by a Variable Force

If the force takes on several successive constant values, we can add the areas of each of the “blocks”:

Example:

Work Done by Varying Force

A force \( \vec{F} = F_x \hat{x} \) varies with \( x \) as shown. Find work done by force on particle that moves from \( x = 0.0 \) m to \( x = 6.0 \) m.

\[
W = A_{\text{total}} = A_1 + A_2 = (5.0 \text{ N})(4.0 \text{ m}) + \frac{1}{2}(5.0 \text{ N})(2.0 \text{ m}) = 20.0 \text{ J} + 5.0 \text{ J} = 25.0 \text{ J}
\]
Work and Springs

The force needed to stretch a spring an amount $x$ is $F = kx$. Therefore, the work done in stretching the spring is

$$W = \frac{1}{2} kx^2 \tag{7-8}$$

Spring Work by Average Force

Force needed to stretch spring distance $x$: $F = kx$

Average force during stretch from $x = 0$ to $x = x_f$: $\frac{1}{2}kx_f$

Work done during stretch:

$$W = F_{\text{Avg}} \Delta x \cos \theta = \left(\frac{1}{2}kx_f\right)x_f = \frac{1}{2}kx_f^2$$

Example: Work Done on Block by Spring

4.0 kg block on frictionless surface is attached to horizontal spring with $k = 400$ N/m. Spring is initially compressed to 5.0 cm. (a) Find work done on block by the spring as block moves from $x = x_1 = -5.0$ cm to its equilibrium position of $x = x_2 = 0$ cm. (b) Find the speed of the block at $x_2 = 0$ cm.
Lecture 15 25/28

\[ W = F_{\text{Avg}} \Delta x \cos \theta = (-\frac{1}{2} k)(-0.05 \text{m})(0.05 \text{m}) = \frac{1}{2}(400 \text{ N/m})(0.05 \text{m})(0.05 \text{m}) = 0.50 \text{ J} \]

\[ W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 \quad \Rightarrow \quad v_f^2 = v_i^2 + \frac{2W}{m} \]

\[ v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(0.50 \text{ J})}{(4.0 \text{ kg})}} = 0.50 \text{ m/s} \]

Lecture 15 26/28

**Power (P)**

Power is a measure of the rate at which work is done. If work \( W \) done during time \( t \):

\[ P = \frac{W}{t} \quad (7-10) \]

SI power unit: 1 J/s = 1 watt = 1 W

Also: 1 horsepower = 1 hp = 746 W

Lecture 15 27/28

**Power**

If an object is moving at a constant speed in the presence of friction, gravity, air resistance, and so forth, the power exerted by the driving force can be written:

\[ P = \frac{F \cdot d}{t} = F \left( \frac{d}{t} \right) = Fv \quad (7-13) \]

Lecture 15 28/28

**TABLE 7-3**

**Typical Values of Power**

<table>
<thead>
<tr>
<th>Source</th>
<th>Approximate power (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hoover Dam</td>
<td>( 1.34 \times 10^9 )</td>
</tr>
<tr>
<td>Car moving at 40 mph</td>
<td>( 7 \times 10^4 )</td>
</tr>
<tr>
<td>Home stove</td>
<td>( 1.2 \times 10^4 )</td>
</tr>
<tr>
<td>Sunlight falling on one square meter</td>
<td>1380</td>
</tr>
<tr>
<td>Refrigerator</td>
<td>615</td>
</tr>
<tr>
<td>Television</td>
<td>200</td>
</tr>
<tr>
<td>Person walking up stairs</td>
<td>150</td>
</tr>
<tr>
<td>Human brain</td>
<td>20</td>
</tr>
</tbody>
</table>
Example: Power of a Motor

A small motor operates a lift that raises a load of bricks weighing 500 N to height of 10 m in 20 s at constant speed. Lift weighs 300 N.

What is the power output of the motor?

\[ W = F \cdot d \cdot \cos \theta = (800 \text{N})(10 \text{m}) = 8000 \text{J} \]

\[ P = \frac{W}{t} = \frac{8000 \text{J}}{20 \text{s}} = 400 \text{ W} \]

\((400 \text{ W} = 0.54 \text{ hp})\)

End of Lecture 15

- Before Friday, read *Walker* 8.1-2
- Homework Assignment #7b should be submitted using WebAssign by 11:00 PM on Sunday, Oct. 11.