Physics 111
Lecture 13 (Walker: Ch. 6.4-5)
Connected Objects
Circular Motion
Centripetal Acceleration
Centripetal Force
Sept. 30, 2009

Midterm Exam 1 on Mon. October 5
(Chapters 1-6; Lectures 1-13)

Example: Connected Blocks
Block of mass $m_1$ slides on frictionless tabletop. It is connected by a string and pulley to a hanging mass $m_2$. Find the acceleration $a$ and string tension $T$.

Example: Atwood’s Machine
Atwood’s Machine consists of two masses connected by a string and pulley. Find the acceleration $a$. 

Free-body diagram for $m_1$

\[ \sum F_{1,x} = T = m_1a \quad \Rightarrow \quad T = m_1a \]

Free-body diagram for $m_2$

\[ \sum F_{2,x} = m_2g - T = m_2a \quad \Rightarrow \quad T = m_2 \left( g - a \right) \]

\[ m_1a = m_2 \left( g - a \right) \quad \Rightarrow \quad m_2g = (m_1 + m_2)a \]

\[ a = g \frac{m_2}{m_1 + m_2} \quad \text{and} \quad T = g \frac{m_1m_2}{m_1 + m_2} \]
\[ \sum F_{1,x} = T - m_1 g = m_1 a \]
\[ \sum F_{2,x} = m_2 g - T = m_2 a \]
\[ T - m_1 g = m_1 a \]
\[ m_2 g - T = m_2 a \]
\[ (m_2 - m_1)g = (m_1 + m_2)a \]
\[ a = g \frac{m_2 - m_1}{m_2 + m_1} \]

**Example: Stagecraft (1)**

200 kg set \( S \) is stored in a loft above a stage. Rope holding set passes up and over a pulley, then is tied backstage. Director tells a 100 kg stagehand to lower the set. He unties and holds on to rope and is hoisted into loft. What is his acceleration?

(\( \text{Assume massless rope \& massless, frictionless pulley.} \))

**Question 3**

\[ \text{a. } T_1 > T_2 \quad \text{b. } T_1 = T_2 \quad \text{c. } T_1 < T_2 \]

**Stagehand:**

\[ \sum (F \text{ on } M) \hat{y} = T \text{ on } M - w_M = T \text{ on } M - m_M g = m_M a_M \]

Set:

\[ \sum (F \text{ on } S) \hat{y} = T \text{ on } S - w_S = T \text{ on } S - m_S g = m_S a_S = -m_S a_M \]

\[ T \text{ on } M = T \text{ on } S = T \]

\[ T - m_M g = m_M a_M \]

\[ T - m_S g = -m_M a_M \]

\[ g(m_S - m_M) = a_M (m_S + m_M) \]

\[ a_M = g \frac{(m_S - m_M)}{(m_S + m_M)} = (9.81 \text{ m/s}^2) \frac{(100 \text{ kg})}{(300 \text{ kg})} = 3.27 \text{ m/s}^2 \]
Uniform Circular Motion

Position: constant
Speed: $|\vec{v}| = \text{constant}$

Particle in Uniform Circular Motion

$T \equiv \text{period} = \text{the time required for one complete rotation.}$

$v = \frac{1 \text{ circumference}}{1 \text{ period}} = \frac{2\pi r}{T}$

For a particle in uniform circular motion, velocity vector $\vec{v}$ is constant in magnitude, but continuously changes direction.
- Particle has acceleration
- There must be a net force on particle

Example: A Rotating Crankshaft

A 4.0 cm diameter crankshaft turns at 2400 rpm. What is speed of a point on surface of the crankshaft?

$$v = \frac{2\pi r}{T} = \frac{2\pi (0.020 \text{ m})}{0.025 \text{ s}} = 5.03 \text{ m/s}$$

Acceleration in Uniform Circular Motion

Object moving at a constant speed $v$ in a circle of radius $r$ has a centripetal (toward center) acceleration $a_{cp}$

- Direction of this acceleration $a_{cp}$ is towards the center of the circle.
- Size of the acceleration is $a_{cp} = \frac{v^2}{r}$
Centripetal Force

- Since an object in uniform circular motion has centripetal acceleration $a_{cp}$, a force is needed to provide this acceleration.
- Force must be provided by rope, friction, gravity, etc.
- Magnitude of force $F_{cp}$ required to keep an object of mass $m$ moving at speed $v$ in a circle of radius $r$, called centripetal force because it points toward the center of the circle, is given by:

$$F_{cp} = ma_{cp} = m \frac{v^2}{r}$$  \hspace{1cm} (6-16)

Sources of Centripetal Force

This centripetal force may be provided by the tension in a string, the normal force, or friction, among other sources.

Example: Rounding a Corner

A car rounds a corner of radius $r$. If the coefficient of friction between tires and road is $\mu_s$, what is the maximum speed $v$ the car can have without skidding?

$$\sum F_x = f_x = \mu F_N = \mu mg$$

$$\sum F_y = N - F_T = N - mg$$

$$\therefore \, \sum F_y = 0 = N - W = N - mg$$

$$\mu mg = \mu N$$

$$v = \sqrt{\frac{\mu mg}{r}}$$

$$v = \sqrt{\frac{0.82 \times 45.0 \, \text{m} \times 9.81 \, \text{m/s}^2}{r}} \approx 19.0 \, \text{m/s}$$

Question: Did we need to know the mass of the car?
If a road is banked at the proper angle $\theta$, a car can round a curve without the assistance of friction between the tires and the road and without skidding. What bank angle $\theta$ is needed for a car of mass $m$ traveling at speed $v$ around a curve of radius $r$?

\[ \sum F_y = 0 = N \cos \theta - W = mg \]
\[ N \cos \theta = mg \]
\[ \sum F_x = N \sin \theta = m a_x = m a_{cp} = \frac{mv^2}{r} \]
\[ \frac{N \sin \theta}{N \cos \theta} = \tan \theta = \frac{mv^2}{r} \cdot \frac{mg}{mg} = \frac{v^2}{gr} \]
\[ \theta = \arctan \frac{v^2}{gr} \]

Notice that there is only one speed at which gravity exactly provides the needed centripetal force.

Centrifuge

A centrifuge is a laboratory device used in chemistry, biology, and medicine for separating out solids suspended in liquid by spinning tubes of the liquid at high speed.

The liquid normal force cannot provide the needed centripetal force on the solids, so the solids collect at the bottom of the tubes.

Tangential & Total Acceleration

An object may be changing its speed (speeding up or slowing down) as it moves in a circular path. In that case, there is tangential acceleration as well as centripetal acceleration.

The total acceleration $a_{total}$ is the vector sum of the centripetal acceleration $a_{cp}$, which points toward the center of rotation, and the tangential acceleration $a_t$, which points in the direction of speed increase.
**Centripetal Acceleration**

\[ a_{cp} = \frac{v^2}{r} \quad \text{(centripetal acceleration)} \]

\[ v_t = \frac{2\pi r}{T} \quad \text{(tangential velocity)} \]

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**Question 2**

Which motion has the largest centripetal acceleration?

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**Example: Spinning in a Circle**

Father places 20 kg child in 5.0 kg cart to which is attached a 2.0 m long rope. He holds the end of the rope and spins the cart and child in a circle, keeping the rope parallel to ground. If tension in rope is 100 N, how many revolutions per minute does cart make?

**Pictorial representation**

**Physical representation**

\[ (F_{net})_r = \sum F_r = T = \frac{MV^2}{r} \]

\[ v_t = \sqrt{\frac{rT}{m}} = \sqrt{\frac{(2.0 \, \text{m})(100 \, \text{N})}{25 \, \text{kg}}} = 2.83 \, \text{m/s} \]

\[ \frac{v_t}{r} = \frac{(2.83 \, \text{m/s})}{(2.0 \, \text{m})} = 1.41 \, \text{rad/s} = 13.5 \, \text{rpm} \]
Example: Satellite's Motion

Satellite moves at constant speed in a circular orbit about center of Earth and near surface of Earth. If magnitude of its acceleration is \( a \equiv g = 9.81 \text{ m/s}^2 \) and Earth's radius is 6,370 km, find its speed \( v \) and time \( T \) required for one complete revolution.

\[
a_{cp} = \frac{v^2}{r} = g
\]

\[
v = \sqrt{rg} = \sqrt{(6,370 \times 10^3 \text{ m})(9.81 \text{ m/s}^2)} = 7.91 \times 10^3 \text{ m/s} = 17,700 \text{ m/h}
\]

\[
T = \frac{2\pi}{v} = \frac{2\pi(6,370 \times 10^3 \text{ m})}{(7.91 \times 10^3 \text{ m/s})} = 5,060 \text{ s} = 84.3 \text{ min}
\]

End of Lecture 13

- Before Friday, read *Walker* Section 7.1
- Homework Assignment #6c should be submitted using WebAssign by 11:00 PM on Saturday, Oct. 3. (HW #6b due tonight.)
- MIDTERM I on Monday October 5. (Chapters 1-6; Lectures 1-13.)