Strings

When you pull on a string or rope, it becomes taut. We say that there is tension in the string.

The tension in a real rope (having mass) will vary along its length, due to the weight of the rope.

For this course, we will assume that all ropes, strings, wires, etc. are massless unless otherwise stated. We will assume tension is the same everywhere in a rope.

Pulleys and Ropes

An ideal pulley is one that simply changes the direction of the tension force.
Pulleys
Strings and ropes often pass over pulleys that change the direction of the tension force. Friction and inertia in the pulley would modify the transmitted tension. We usually assume that pulleys are **massless** and **frictionless**.

Springs
A compressed spring exerts a pushing force on an object. A stretched spring exerts a pulling force on an object.

**Spring Force**
A spring can either push (when compressed) or pull (when stretched).

**Hooke's Law for Springs**
Hooke's law states that spring force is proportional to the amount $x_{st}$, that the spring is stretched or compressed:

$$ F_{spring} = -kx_{st} $$

The constant $k$ is called the **spring constant**. In the SI system, $k$ has units of N/m or kg/s².
Spring Force

Direction

The direction of spring force is that which opposes the compression or extension of the spring.

\[ F_{\text{spring}} = -kx_{\text{str}} \]

Spring Example

One end of a spring of length 0.2m is attached to the ceiling. When mass of 2 kg is hung from the other end of the spring, the spring stretches to 0.25 m length.

(a) What is the spring constant of the spring?

\[ F_{sp} = mg = kx_{\text{stretch}} = k(0.25m - 0.2m) \]

\[ k = \frac{mg}{0.05m} = \frac{(2kg)(9.8 \text{ N/kg})}{0.05m} \]

= 390 N/m

(b) How long would spring be if 4 kg mass hung from it?

\[ F_{sp} = kx_{\text{str}} = mg; \quad x_{\text{str}} = \left(4kg\right)(9.8 \text{N/kg})/(390 \text{N/m}) \]

\[ x_{\text{str}} = 0.1 \text{m and spring length is 0.3m} \]

Force Misconceptions

Is there a "force of motion" or a "force of inertia" that keeps a moving object moving? No.

Can a normal or tension force act "sideways"? No.

Remember that:

1. Every force must have an agent that produces it;
2. Every contact force must act only at the point of contact;
3. The normal force acts only perpendicular to the surface in contact;
4. The friction force acts only parallel to the surface in contact;
5. The tension force from a string or rope acts only along the line of the string or rope.

Translational Equilibrium

When an object is in translational equilibrium (has no acceleration), the net force on it is zero:

\[ \sum \vec{F} = 0 \]

\[ \sum F_x = 0 \quad \sum F_y = 0 \]

This allows the calculation of unknown or unmeasured forces.
Translational Equilibrium

Person lifts bucket of water from bottom of well with constant speed \( v \). Because speed is constant, acceleration must be zero, and net force on bucket is zero, so \( T_1 = W \).  

**Q1:** What is \( T_2 \) in terms of \( W \)?

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**Question 2**

In which situation is the tension on the rope larger?

- a. A
- b. B
- c. Both tensions are the same

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**Example: Mountain Climbing (1)**

90 kg mountain climber is suspended from ropes as shown. Rope 3 can have a maximum tension of 1500 N before breaking. Smallest angle \( \theta \) before rope breaks?

**Pictorial representation**

**Physical representation**

\[
\begin{align*}
\sum (F_{on M})_y &= T_{K on C} - w = T_{K on C} - mg = 0 \\
\sum (F_{on K})_x &= T_3 - T_1 \cos \theta = 0 \\
\sum (F_{on K})_y &= T_1 \sin \theta - T_{C on K} = 0 \\
T_{K on C} &= T_{C on K} = mg \\
T_1 \cos \theta &= T_3 \\
T_1 \sin \theta &= mg \\
\text{so } \tan \theta &= mg / T_3 \\
\theta_{max} &= \tan^{-1}(mg / T_{3max}) \\
&= \tan^{-1}[(90 \text{ kg})(9.81 \text{ m/s}^2) / (1500 \text{ N})] = 30.5^\circ
\end{align*}
\]
Connected Objects

When forces are exerted on connected objects, their accelerations are the same.

If there are two objects connected by a string, and we know the force and the masses, we can find the acceleration and the tension:

Example: Comparing Tensions

Blocks A and B are connected by massless String 2 and pulled across frictionless surface by massless String 1. The mass of B is larger than mass of A. Is tension in String 2 smaller, equal, or larger than tension in String 1?

Blocks must be accelerating to the right, because there is a net force in that direction. We use the massless string approximation to directly relate the string tensions on A and B due to String 2:

\[ T_{A \text{ on } B} = T_{B \text{ on } A} \]
\[ (F_{A \text{ net}})_x = T_1 - T_{B \text{ on } A} \]
\[ = T_1 - T_2 = m_A a_A x \]

so \[ T_1 = T_2 + m_A a_A x \]

Therefore, \[ T_1 > T_2 \]
**Connected Objects**

If there is a pulley, it is easiest to have the coordinate system follow the string:

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**Example: Connected Blocks**

Block of mass $m_1$ slides on frictionless tabletop. It is connected by a string and pulley to a hanging mass $m_2$. Find the acceleration $a$ and string tension $T$.

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**Example: Atwood’s Machine**

Atwood’s Machine consists of two masses connected by a string and pulley. Find the acceleration $a$.

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\[
\begin{align*}
\sum F_{1,x} &= T = m_1 a \
\sum F_{2,x} &= m_2 g - T = m_2 a \\
&\Rightarrow \quad T = m_1 a \\
&\Rightarrow \quad T = m_2 (g - a) \\
&\Rightarrow \quad m_1 a = m_2 (g - a) \\
&\Rightarrow \quad m_2 g = (m_1 + m_2) a \\
&\Rightarrow \quad a = g \frac{m_2}{m_1 + m_2} \\
&\quad \text{and} \quad T = g \frac{m_1 m_2}{m_1 + m_2}
\end{align*}
\]
### Free-body diagram for $m_1$

\[ \sum F_{1,x} = T - m_1 g = m_1 a \]

\[ T - m_1 g = m_1 a \]

\[ m_2 g - T = m_2 a \]

\[ (m_2 - m_1) g = (m_1 + m_2) a \]

\[ a = g \frac{m_2 - m_1}{m_2 + m_1} \]

### Free-body diagram for $m_2$

\[ \sum F_{2,x} = m_2 g - T = m_2 a \]

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### Question 3

a. $T_1 > T_2$

b. $T_1 = T_2$

c. $T_1 < T_2$

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### End of Lecture 12

- Before Wednesday, read *Walker* 6.4-5
- Homework Assignment #6b should be submitted using WebAssign by 11:00 PM on Wednesday, Sept. 30.