Physics 111  
Lecture 7 (Walker: 4.2-5)  
2D Motion Examples  
Projectile Motion  
Sept. 16, 2009

2-D Motion -- Constant Acceleration

\[
\begin{align*}
\vec{r} &= \vec{r}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2 \\
\vec{v} &= \vec{v}_0 + \vec{a} t \\
x(t) &= x_0 + v_{0x} t + \frac{1}{2} a_x t^2 \\
y(t) &= y_0 + v_{0y} t + \frac{1}{2} a_y t^2 \\
\end{align*}
\]

Example: Hummer Accelerates

Hummingbird is initially moving vertically with speed of 4.6 m/s and accelerating horizontally at a constant rate of 11 m/s².

Find the horizontal and vertical distance through which it moves in 0.55 s.

\[
\begin{align*}
x &= x_0 + v_{0x} t + \frac{1}{2} a_x t^2 \\
y &= y_0 + v_{0y} t + \frac{1}{2} a_y t^2 \\
\end{align*}
\]

Projectile Motion  
(Objects with vertical acceleration & constant horizontal velocity)

Assumptions:
- Ignore air resistance
- Use \( g = 9.80 \text{ m/s}^2 \), direction downward, for free fall
- Ignore the Earth's rotation

If the \( y \)-axis points upward, the acceleration in the \( x \)-direction is zero and the acceleration in the \( y \)-direction is \(-9.80 \text{ m/s}^2\) during the projectile motion.
Equations: Constant Acceleration with $a_x = 0$ and $a_y = -g$

\[ x(t) = x_0 + v_{0x} t \]
\[ v_x = v_{0x} \]
\[ y(t) = y_0 + v_{0y} t - \frac{1}{2} gt^2 \]
\[ v_y = v_{0y} - gt \]
\[ v_y^2 = v_{0y}^2 - 2g\Delta y \]

Projectile Motion: Basic Equations

The acceleration is independent of direction of velocity:

Independence of Vertical and Horizontal Motion

When you drop a ball while running with constant velocity, it appears to you to drop straight down from the point where you released it. To a person at rest, the ball follows a curved path that combines horizontal and vertical motions.

Launch Angle

Launch angle: direction of initial velocity with respect to horizontal
Zero Launch Angle

In the zero launch angle case, the initial velocity in the $y$-direction is zero. Here are the equations of motion, with $x_0 = 0$ and $y_0 = h$:

\[
\begin{align*}
x &= v_0 t \\
y &= h - \frac{1}{2} g t^2 \\
v_x &= v_0 = \text{constant} \\
v_y &= -g t
\end{align*}
\]

Example: Cliff Diving

George and Sam dive from a high overhanging cliff into a lake. George drops straight down. Sam runs horizontally and dives outward.

(a) If they leave the cliff at the same time, which boy reaches the water first?

Since vertical motion determines time to reach the water, both boys reach the water at the same.

(b) Which boy hits the water with the greater speed?

George reaches water with only vertical velocity; Sam reaches the water with both horizontal and vertical velocity components. Vertical velocities same for both, so Sam’s speed at water greater than George’s.

Example: Electron in Motion

Electron traveling with horizontal speed of $2.10 \times 10^9$ cm/s goes between deflection plates that give it an upward acceleration of $5.30 \times 10^{17}$ cm/s$^2$. After that,

(a) How long does it take for electron to cover a horizontal distance of 6.20 cm?

(b) What is the vertical displacement during this time?

\[
\begin{align*}
x(t) &= x_0 + v_{0x} t + \frac{1}{2} a_x t^2 = v_{0x} t \\
t &= \frac{x}{v_{0x}} = \frac{(6.20 \text{ cm})}{(2.10 \times 10^9 \text{ cm/s})} = 2.95 \times 10^{-9} \text{ s} \\
y(t) &= y_0 + v_{0y} t + \frac{1}{2} a_y t^2 = \frac{1}{2} a_y t^2 \\
y &= \frac{1}{2} a_y t^2 = \frac{1}{2} (5.30 \times 10^{17} \text{ cm/s}^2)(2.95 \times 10^{-9} \text{ s})^2 = 2.31 \text{ cm}
\end{align*}
\]
**Projectile - General Launch Angle**

In general, \( v_{0x} = v_0 \cos \theta \) and \( v_{0y} = v_0 \sin \theta \)

(This ASSUMES \( \theta \) is measured CCW from +x axis!)

The equations of motion are:

\[
x = x_0 + v_{0x}t
\]

\[
y = y_0 + v_{0y}t - \frac{1}{2}gt^2
\]

\[v_x = v_0 \cos \theta\]

\[v_y = v_0 \sin \theta - gt\]

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**Example: A Home Run**

Baseball leaves bat making 30° angle with ground. It crosses fence 100 m from home plate at same height that it was struck. (Neglect air resistance.)

What was its velocity as it left bat?

**Known**

- \( x_0 = y_0 = 0 \)
- \( \theta = 30^\circ \)
- \( t_0 = 0 \) s
- \( x_1 = 100 \) m
- \( y_1 = 0 \) m

**Find**

\( v_0 \)

**Pictorial representation**

\[
v_{0x} = v_0 \cos \theta = 0.866 v_0
\]

\[
x_1 = 100 \text{ m} = 0.866 v_0 t_1 \quad \text{so} \quad t_1 = \frac{100 \text{ m}}{0.866 v_0}
\]

\[
v_{0y} = v_0 \sin \theta = 0.5 v_0
\]

\[
y_1 = 0 = v_0 t_1 - \frac{1}{2} gt_1^2
\]

\[
= (0.5v_0)[(100 \text{ m}/(0.866v_0)] - (4.9 \text{ m/s}^2)[100 \text{ m}/(0.866v_0)]^2
\]

Thus 57.7 m = \((65330 \text{ m}^3/\text{s}^2)/v_0^2\)

\[
v_0 = 33.6 \text{ m/s}
\]
Example: Catch a Thief
Police officer chases thief across rooftops. They are both running when they come to a gap between buildings that is 4.0 m wide and has a drop of 3.0 m. Thief, having studied physics, leaps at 5.0 m/s at an angle of 45° above horizontal and clears gap. Officer did not study physics and thinks he should maximize his horizontal velocity, so leaps horizontally at 5.0 m/s. (a) Does he clear the gap? (b) By how much does the thief clear the gap?

\[ y = y_0 + v_{0y}t - \frac{1}{2}gt^2 \]

Police: \[ y = 0 + 0 - \frac{1}{2}(9.81 \text{ m/s}^2)t^2 \]
\[ t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2 \times 3.0}{9.81}} \approx 0.782 \text{ s} \]
\[ x = x_0 + v_{0x}t = 0 + (5.0 \text{ m/s})(0.782 \text{ s}) \approx 3.91 \text{ m} \]

Thief: \[ y = 0 + (5.0 \text{ m/s})\sin 45^\circ t - \frac{1}{2}(9.81 \text{ m/s}^2)t^2 \]
\[ t = \frac{1}{2} \text{ or } t = 1.22 \text{ s} \]
\[ x = x_0 + v_{0x}t = 0 + (5.0 \text{ m/s})(1.22 \text{ s})\cos 45^\circ \approx 4.31 \text{ m} \]

No!

Projectile Motion: Range

\[ R = \left(\frac{v_0^2}{g}\right) \sin 2\theta \]

The range is a maximum when \( \theta = 45^\circ \):

\[ R_{\text{max}} = \frac{v_0^2}{g} \]
ConcepTest

A gun is accurately aimed at criminal hanging from gutter of building. The target is well within gun’s range, but the instant gun is fired and bullet moves with a speed $v_o$, criminal lets go and drops to ground. What happens? The bullet --

1. hits criminal regardless of value of $v_o$.
2. hits criminal only if $v_o$ is large enough.
3. misses criminal.

Example: A Supply Drop

Helicopter drops supply package to flood victims on raft. When package is released, helicopter is 100 m directly above raft and flying at velocity 25.0 m/s at angle 36.9° above horizontal. [Neglect air resistance].

(a) How long is package in air?
(b) How far from raft does package land?
\[ v_{0y} = v_0 \sin \theta_0 = (25.0 \text{ m/s})(\sin 36.9^\circ) = 15.0 \text{ m/s}, \quad v_{0x} = v_0 \cos \theta_0 = 20.0 \text{ m/s} \]

\[ y(t) = v_{0y}t - \frac{1}{2} gt^2 \]

\[-100 \text{ m} = (15.0 \text{ m/s})t - (0.5)(9.8 \text{ m/s}^2)t^2 \]

\[ t = \frac{(15.0 \text{ m/s}) \pm \sqrt{(15.0 \text{ m/s})^2 - 2(9.81 \text{ m/s}^2)(-100 \text{ m})}}{(9.81 \text{ m/s}^2)} \]

\[ t = -3.24 \text{ s and } t = 6.30 \text{ s} \]

\[ x = x_h = v_{0x}t = (20.0 \text{ m/s})(6.30 \text{ s}) = 126 \text{ m} \]

The package will hit the water 126 m from raft.

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**Summary - 2D Kinematics**

- Components of motion in the x- and y-directions can be treated independently.
- In projectile motion, the y-component of acceleration is \(-g\); there is no x- acceleration.
- If the launch angle is zero, the initial velocity has only an x-component.
- The path followed by a projectile is a parabola.
- The range is the horizontal distance the projectile travels.

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**End of Lecture 7**

- Before the next lecture, *Walker* 5.1-3
- Homework Assignment #4a should be submitted using WebAssign by 11:00 PM on Saturday, Sept. 19.