Physics 111

Lecture 6 (Walker: 3.6,4.1-2)

Relative Motion

2D Motion Basics

Sept. ***14, 2009***

Vector Motion with Constant Acceleration

Average velocity:

$$\vec{v}_{av} = \tfrac{1}{2}(\vec{v}_0 + \vec{v})$$

Velocity as a function of time:

$$\vec{v}(t) = \vec{v}_0 + \vec{a}t$$

Position as a function of time:

$$\vec{r}(t) = \vec{r}_0 + \vec{v}_{av}t = \vec{r}_0 + \tfrac{1}{2}(\vec{v}_0 + \vec{v})t$$

$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2$$

Relative Motion

Suppose you swim downstream at 3 mi/hr relative to the water and the water current is moving at 2 mi/hr. What is your speed relative to the bank of the river?

Velocities must be added as vectors.

$\vec{\mathbf{v}}_{sw} = 3 \text{ mi/hr } \hat{x}$ + $\vec{\mathbf{v}}_{wb} = 2 \text{ mi/hr } \hat{x}$ = $\vec{\mathbf{v}}_{sb} = 5 \text{ mi/hr } \hat{x}$

velocity of swimmer relative to water

velocity of water relative to bank

velocity of swimmer relative to bank

Relative Motion

In general

$$\vec{\mathbf{v}}_{13} = \vec{\mathbf{v}}_{12} + \vec{\mathbf{v}}_{23}$$

- $\vec{\mathbf{v}}_{13}$: velocity of 1 relative to 3
- $\vec{\mathbf{v}}_{12}$: velocity of 1 relative to 2
- $\vec{\mathbf{v}}_{23}$: velocity of 2 relative to 3

Note that $\vec{\mathbf{v}}_{12} = -\vec{\mathbf{v}}_{21}$

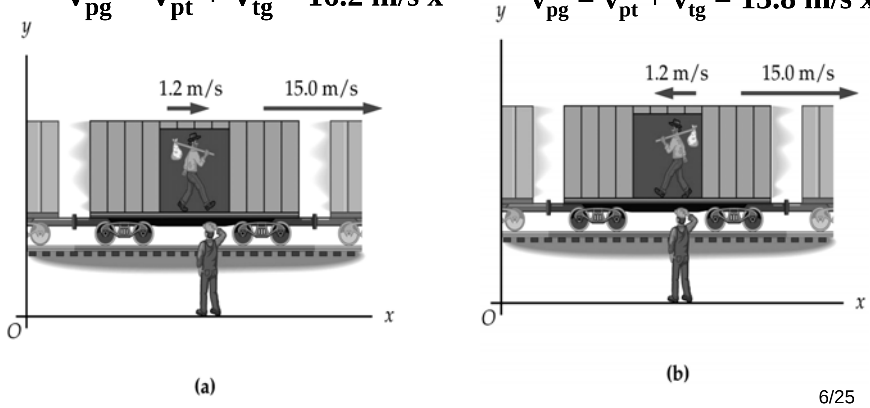
Example

A man is walking toward the back of a train at 2 m/s relative to the train, which is heading North. His velocity relative to the ground is 1.1 m/s North. What is the velocity of the train relative to the ground?

Relative Motion

The speed of a passenger with respect to the ground depends on the relative directions of the passenger's and train's speeds:

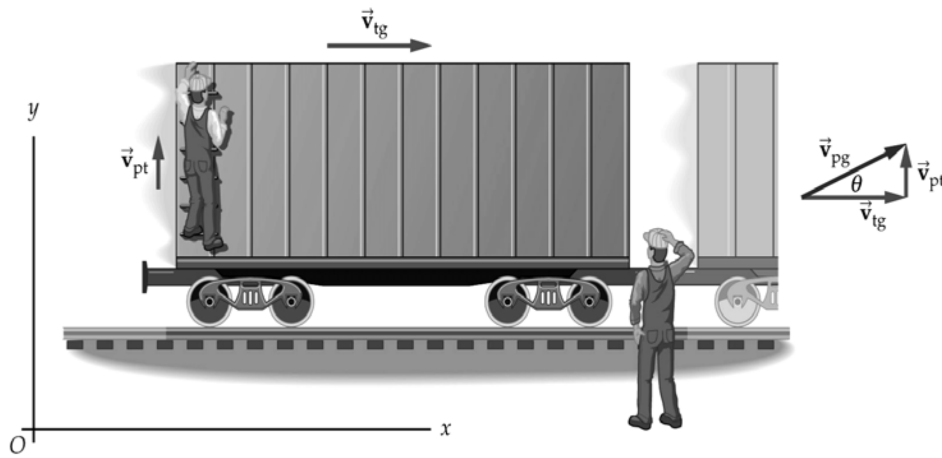
$\vec{v}_{pg} = \vec{v}_{pt} + \vec{v}_{tg} = 16.2 \text{ m/s } \hat{x}$ $\vec{v}_{pg} = \vec{v}_{pt} + \vec{v}_{tg} = 13.8 \text{ m/s } \hat{x}$



Relative Motion

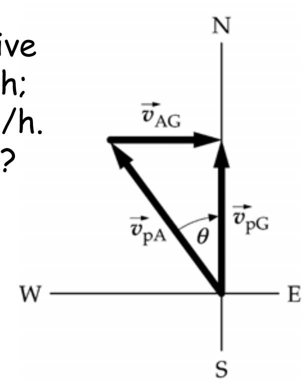
$$\vec{\mathbf{v}}_{pg} = \vec{\mathbf{v}}_{pt} + \vec{\mathbf{v}}_{tg}$$

This also works in two dimensions:



Example: Flying a Plane

Pilot wishes to fly plane due north relative to ground. Airspeed of plane is 200 km/h; wind blowing from west to east at 90 km/h.
(a) In which direction should plane head?
(b) What will be ground speed of plane?



$$\vec{v}_{pG} = \vec{v}_{pA} + \vec{v}_{AG}$$

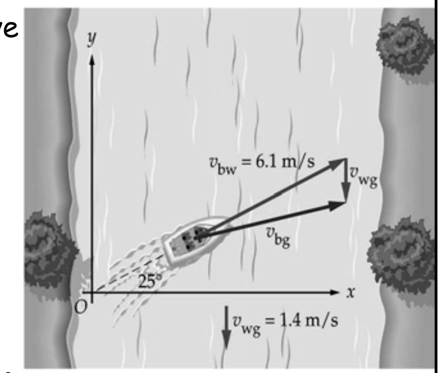
$$\theta = \arcsin\frac{v_{AG}}{v_{pA}} = \arcsin\frac{(90\text{ km/h})}{(200\text{ km/h})} = 26.7° \text{ west of north}$$

$$v_{pG} = \sqrt{v_{pA}^2 - v_{AG}^2} = \sqrt{(200\text{ km/h})^2 - (90\text{ km/h})^2} = 179\text{ km/h}$$

Example: Crossing a River

You are in a boat with speed relative to water of v_{bw} = 6.1 m/s. Boat points at angle θ = 25° upstream on river flowing at v_{wg} = 1.4 m/s.

(a) What are your speed v_{bg} and angle θ_{bg} relative to the ground?



$$\vec{v}_{bg} = \vec{v}_{bw} + \vec{v}_{wg}$$

$$\vec{v}_{wg} = (-1.4\ \text{m/s})\,\hat{y}$$

$$\vec{v}_{bw} = (6.1\ \text{m/s})\cos 25°\,\hat{x} + (6.1\ \text{m/s})\sin 25°\,\hat{y}$$
$$= (5.5\ \text{m/s})\hat{x} + (2.6\ \text{m/s})\hat{y}$$

$$v_{bg} = \sqrt{(5.5\ \text{m/s})^2 + (1.2\ \text{m/s})^2} = 5.6\ \text{m/s}$$

$$\vec{v}_{bg} = (5.5\ \text{m/s})\hat{x} + (2.6\ \text{m/s} - 1.4\ \text{m/s})\hat{y}$$
$$= (5.5\ \text{m/s})\hat{x} + (1.2\ \text{m/s})\hat{y}$$

$$\theta_{bg} = \tan^{-1}\left[(1.2\ \text{m/s}) / (5.5\ \text{m/s})\right] = 12°$$

Motion in Two Dimensions

Motion in the x- and y-directions should be solved ***separately***.

TABLE 4–1 Constant-Acceleration Equations of Motion

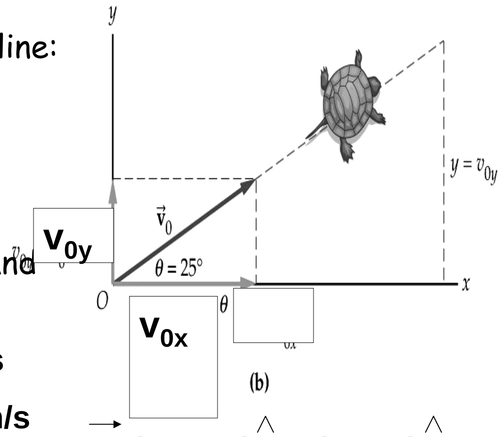
Position as a function of time	Velocity as a function of time	Velocity as a function of position
$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$	$v_x = v_{0x} + a_x t$	$v_x^2 = v_{0x}^2 + 2a_x\Delta x$
$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$	$v_y = v_{0y} + a_y t$	$v_y^2 = v_{0y}^2 + 2a_y\Delta y$

Constant Velocity Case (a_x=0; a_y=0)

If velocity is constant,
motion is along a straight line:

$$x(t) = x_0 + v_{0x}t;$$

$$y(t) = y_0 + v_{0y}t$$



Example: Turtle walks from origin at speed v_0=2.0 m/s and angle 25^0. Position at t=5s?

$v_x = v_{0x} = v_0\cos(25°) = 1.81$ m/s

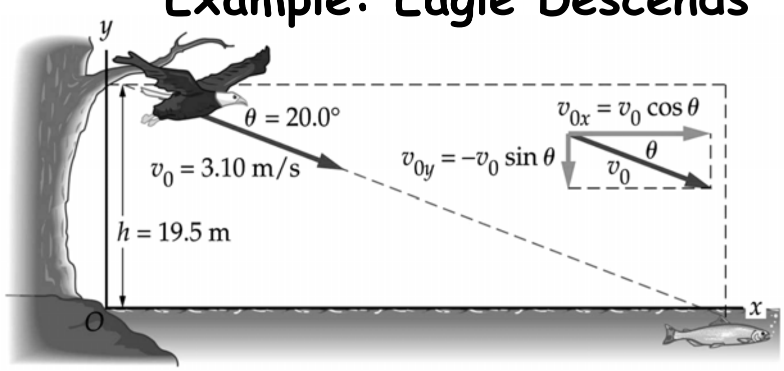
$v_y = v_{0y} = v_0\sin(25°) = 0.84$ m/s

$x = x_0 + v_x t = (1.81\text{m/s})(5\text{s}) = 9.05$m

$y = y_0 + v_y t = (0.84\text{m/s})(5\text{s}) = 4.2$ m

$\vec{r} = (9.05\text{ m})\,\hat{x} + (4.2\text{ m})\,\hat{y}$

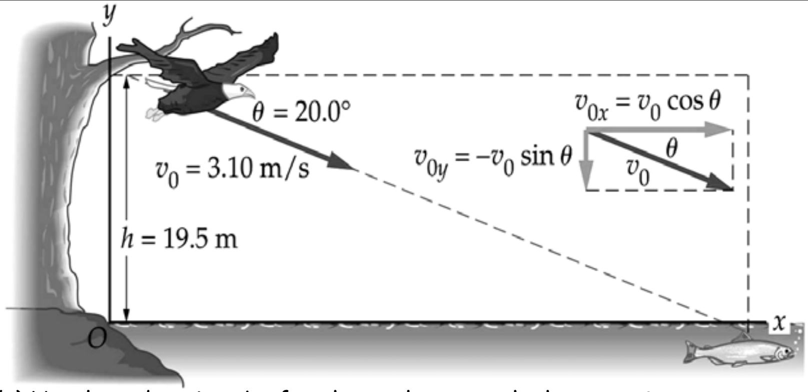
Example: Eagle Descends



Eagle perched on tree limb 19.5 m above water spots fish. He pushes off limb and descends toward water, maintaining constant speed of 3.20 m/s at 20° below horizontal.

(a)How long does it take for eagle to reach water?

(b)How far has eagle traveled horizontally when it reaches water?



(a) How long does it take for the eagle to reach the water?

(b) How far has the eagle traveled horizontally when it reaches the water?

$$v_{0x} = v_0 \cos\theta = (3.10 \text{ m/s})\cos 20° = 2.91 \text{ m/s}$$

$$v_{0y} = v_0 \sin\theta = (3.10 \text{ m/s})\sin 20° = -1.06 \text{ m/s}$$

$$t = h / v_{0y} = (-19.5 \text{ m})/(-1.06 \text{ m/s}) = 18.4 \text{ s}$$

$$x = x_0 + v_{0x}t = (2.91 \text{ m/s})(18.4 \text{ s}) = 53.5 \text{ m}$$

Constant Acceleration Case

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2$$

$$x(t) = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

$$y(t) = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$v_x = v_{0x} + a_x t$$

$$v_y = v_{0y} + a_y t$$

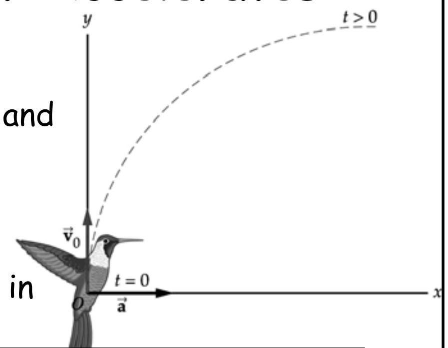
$$v_x^2 = v_{0x}^2 + 2a_x \Delta x$$

$$v_y^2 = v_{0y}^2 + 2a_y \Delta y$$

Example: Hummer Accelerates

Hummingbird is initially moving vertically with speed of 4.6 m/s and accelerating horizontally at a constant rate of 11 m/s^2.

Find the horizontal and vertical distance through which it moves in 0.55 s.



$$x = \cancel{x_0} + \cancel{v_{0x}}t + \tfrac{1}{2}a_x t^2$$

$$y = \cancel{y_0} + v_{0y}t + \tfrac{1}{2}\cancel{a_y}t^2$$

Projectile Motion
(Objects with vertical acceleration & constant horizontal velocity)

Assumptions:

- Ignore air resistance
- Use g = 9.80 m/s^2, direction downward, for free fall
- Ignore the Earth's rotation

If the y-axis points upward, the acceleration in the x-direction is zero and the acceleration in the y-direction is –9.80 m/s^2 during the projectile motion.

Equations: Constant Acceleration with $a_x = 0$ and $a_y = -g$

$$x(t) = x_0 + v_{0x}t \qquad v_x = v_{0x}$$

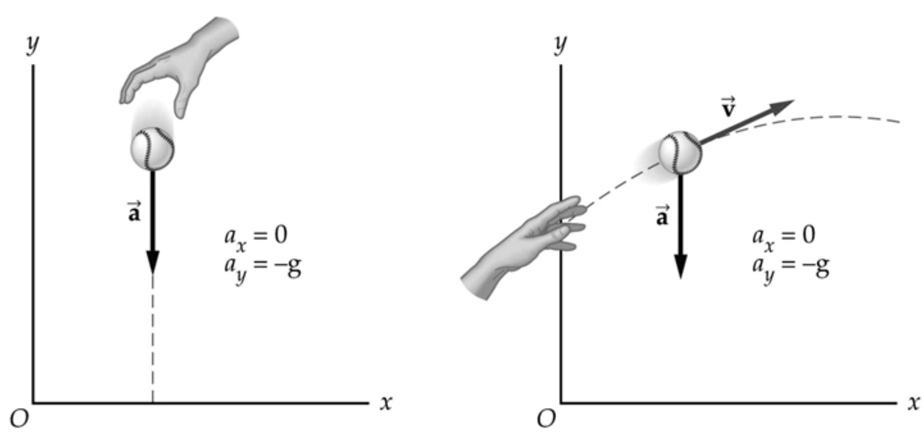
$$y(t) = y_0 + v_{0y}t - \tfrac{1}{2}gt^2 \qquad v_y = v_{0y} - gt$$

$$v_x^2 = v_{0x}^2$$

$$v_y^2 = v_{0y}^2 - 2g\Delta y$$

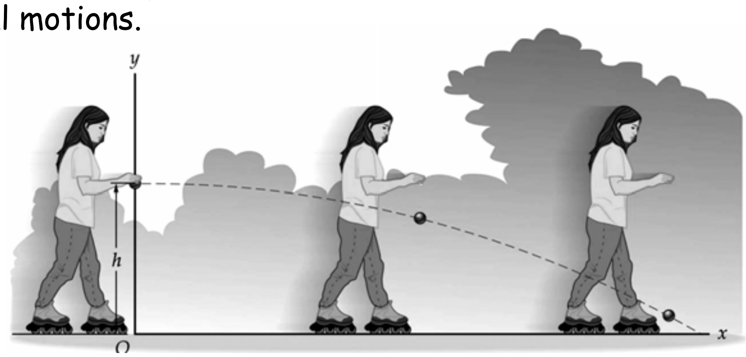
Projectile Motion: Basic Equations

The acceleration is independent of direction of velocity:



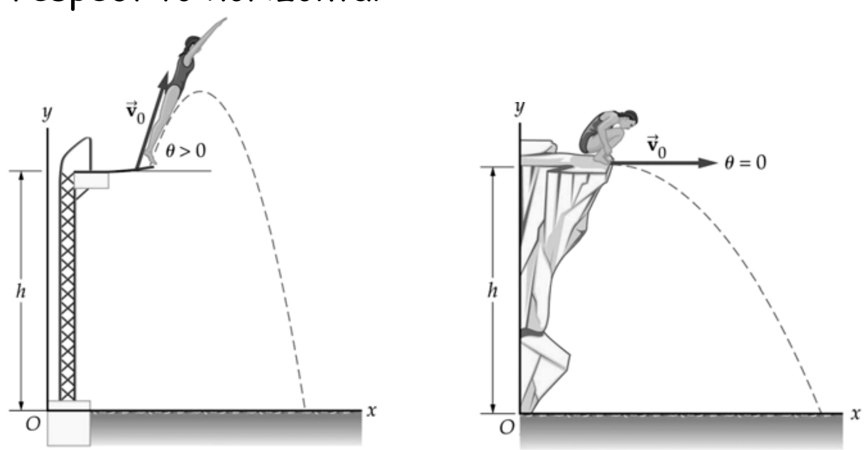
Independence of Vertical and Horizontal Motion

When you drop a ball while running with constant velocity, it appears to you to drop straight down from the point where you released it. To a person at rest, the ball follows a curved path that combines horizontal and vertical motions.



Zero Launch Angle

Launch angle: direction of initial velocity with respect to horizontal



Zero Launch Angle

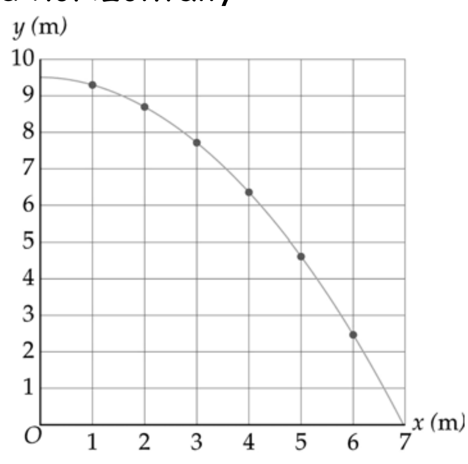
In the zero launch angle case, the initial velocity in the y-direction is zero. Here are the equations of motion, with $\mathbf{x_0} = \mathbf{0}$ and $\mathbf{y_0} = \mathbf{h}$:

$$x = v_0 t$$
$$y = h - \frac{1}{2} g t^2$$

$$v_x = v_0 = \text{constant} \qquad {v_x}^2 = {v_0}^2 = \text{constant}$$
$$v_y = -gt \qquad {v_y}^2 = -2g\Delta y$$

Zero-Launch Trajectory

This is the ***trajectory*** of a projectile launched horizontally:



Example: Cliff Diving

George and Sam dive from a high overhanging cliff into a lake. George drops straight down. Sam runs horizontally and dives outward.

(a) It they leave the cliff at the same time, which boy reaches the water first?

Since vertical motion determines time to reach the water, both boys reach the water at the same.

(b) Which boy hits the water with the greater speed?

George reaches water with only vertical velocity; Sam reaches the water with both horizontal and vertical velocity components. Vertical velocities same for both, so Sam's speed at water greater than George's.

Example: Electron in Motion

Electron traveling with horizontal speed of 2.10×10^9 cm/s goes between deflection plates that give it an upward acceleration of 5.30×10^{17} cm/s^2. After that,

(a) How long does it take for electron to cover a horizontal distance of 6.20 cm?

(b) What is the vertical displacement during this time?

$$x(t) = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2 = v_{0x}t$$

$$t = \frac{x}{v_{0x}} = \frac{(6.20\ \text{cm})}{(2.10\times10^9\ \text{cm/s})} = 2.95\times10^{-9}\ \text{s}$$

$$y(t) = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2 = \tfrac{1}{2}a_y t^2$$

$$y = \tfrac{1}{2}a_y t^2 = \tfrac{1}{2}(5.30\times10^{17}\ \text{cm/s}^2)(2.95\times10^{-9}\ \text{s})^2 = 2.31\ \text{cm}$$

End of Lecture 6

- Before the next lecture, read <u>*Walker*</u> 4.3-5
- Homework Assignment #3b should be submitted using WebAssign by 11:00 PM on Thursday, Sept. 17.