

**Physics 111**  
***Lecture 30 (Walker: 15.6-7)***  
**Fluid Dynamics**  
**April 15, 2009**

***Midterm #2 - Monday April 20***  
***Chap. 7 ,Chap. 8 (not 8.5)***  
***Chap. 9 (not 9.6, 9.8)***  
***Chap. 10, Chap. 11 (not 11.8-9)***  
***Chap. 13 (not 13.6-8)***  
***Chap. 15 (not 15.8-9)***

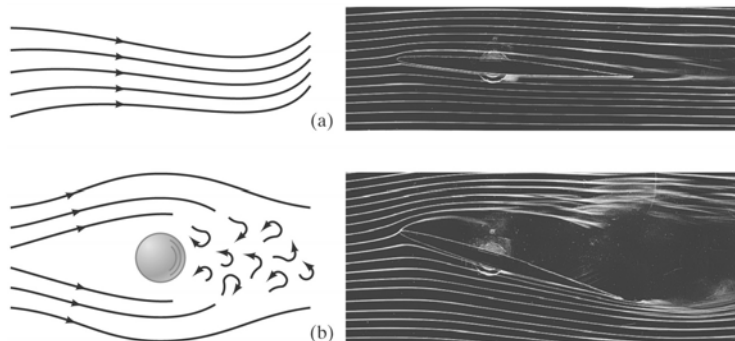
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## Fluids in Motion: Flow and Continuity

If the flow of a fluid is smooth, it is called streamline or laminar flow (a). We will work with laminar flow.

Above a certain speed, the flow becomes turbulent (b). Turbulent flow has eddies; the viscosity (friction) of the fluid is much greater when eddies are present.



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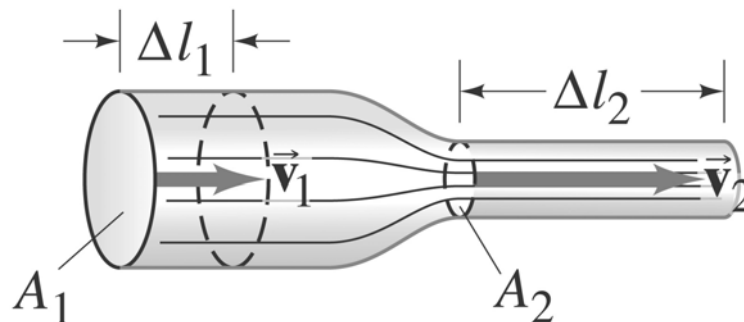
## Flow Rate and the Equation of Continuity

Look at fluid flowing in a tube or pipe.

The mass flow rate is the mass of fluid that passes a given point per unit time. The mass flow rates at any two points in a pipe or tube must be equal, as long as no fluid is being added or taken away.

“What goes in must come out”

## Volume Flow Rate



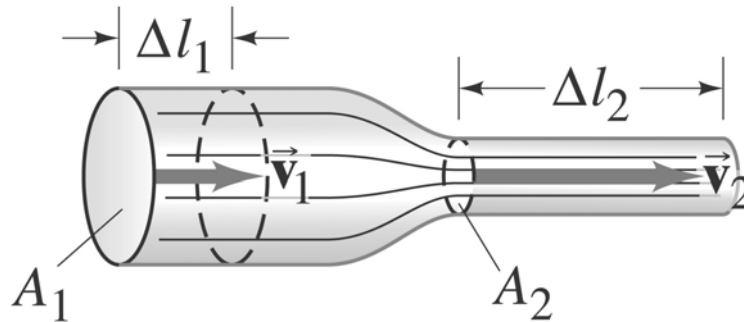
$\Delta V_1$ , Volume of fluid flowing through  $A_1$  in time  $\Delta t$ , is  $A_1 \Delta l_1$  where  $\Delta l_1$  is the distance fluid flows in  $\Delta t$ . If fluid in region 1 moving at speed  $v_1$ ,  $\Delta l_1 = v_1 \Delta t$  and  $\Delta V_1 = A_1 v_1 \Delta t$ .

Volume flow rate  $\Delta V_1 / \Delta t = A_1 v_1$ .

## Mass Flow Rate

The mass flow rate at point 1 is

$$\Delta M_1 / \Delta t = \rho_1 \Delta V_1 / \Delta t = \rho_1 A_1 v_1$$



Must have same mass flow rate at point 2:

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 \quad \text{General equation of continuity.}$$

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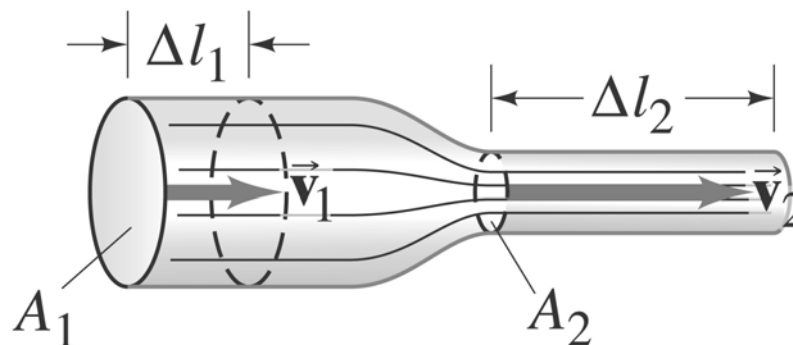
## Flow Rate and the Equation of Continuity

If the density doesn't change – typical for liquids – this simplifies to  $A_1 v_1 = A_2 v_2$ .

Where the pipe is wider, the flow is slower.

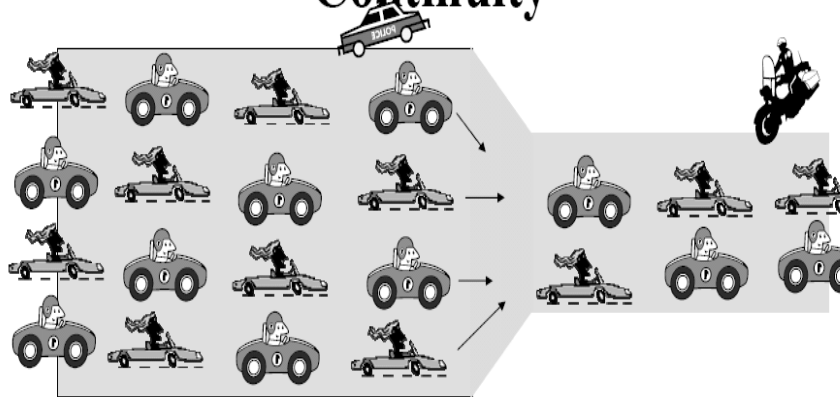
$$A_1 v_1 = A_2 v_2$$

Equation of Continuity  
for Liquids.



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# Continuity



Four-lane highway merges to two-lane. Officer in police car observes 8 cars passing per second, at 30 mph. How many cars does officer on motorcycle observe passing per second?  
 A) 4 B) 8 C) 16

How fast must cars in two-lane section be going?  
 A) 15 mph B) 30 mph C) 60 mph

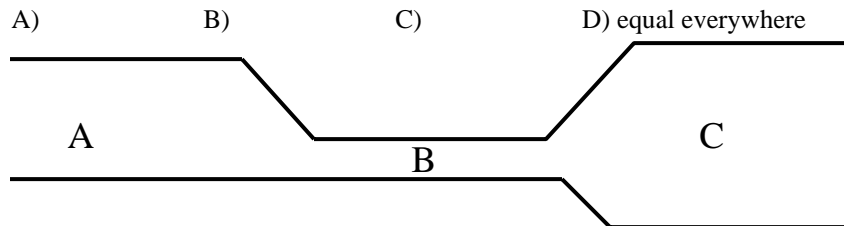
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# Water Pipe

Water is flowing continuously in the pipe shown below. Where is the velocity of the water greatest?

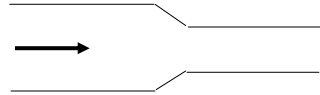


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## Example

A horizontal pipe contains water at a pressure of 110 kPa flowing with a speed of 1.4 m/s. When the pipe narrows to one-half its original radius, what is the speed?



$$v_f A_f = v_i A_i$$

$$v_f = v_i A_i / A_f = v_i [\pi r_i^2] / [\pi (r_i/2)^2] = 4v_i$$

$$v_f = 5.6 \text{ m/s}$$

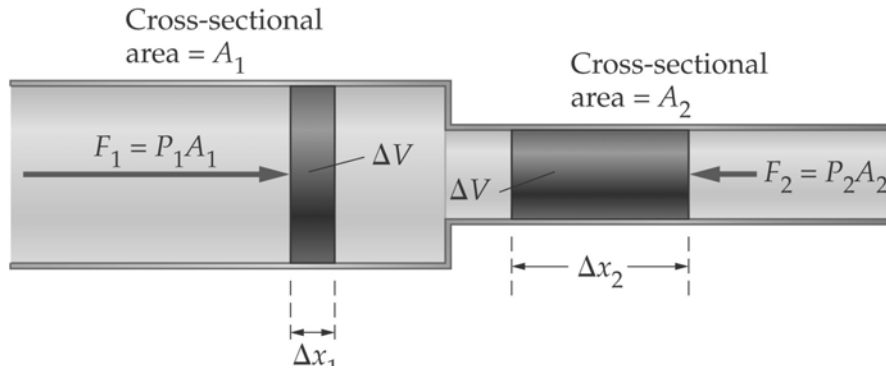
## Bernoulli's Equation

**Bernoulli related the pressure in a fluid to fluid kinetic and potential energies.**

**Consider a volume element  $\Delta V$  of fluid moving through a pipe. For now, we neglect fluid friction (we assume fluid has no viscosity).**

## Bernoulli's Equation

When a fluid moves from a wider area of a pipe to a narrower one, its speed increases; therefore, work has been done on it.



$$\Delta W_{\text{total}} = (P_1 - P_2) \Delta V$$

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## Bernoulli's Equation - Const. Height

The kinetic energy of a fluid element is:

$$K = \frac{1}{2}(\Delta m)v^2 = \frac{1}{2}(\rho \Delta V)v^2$$

Equating the work done on fluid element to its increase in kinetic energy gives:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

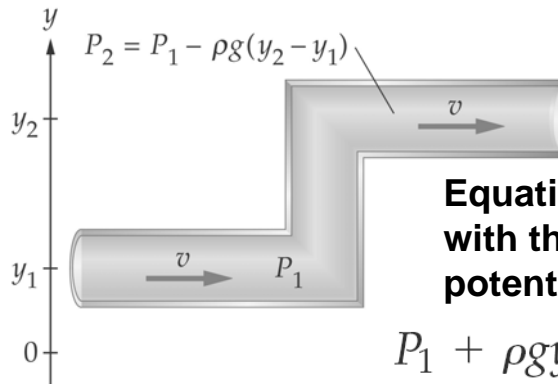
This is Bernoulli's Equation for a pipe at constant height.

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## Bernoulli's Equation - Const. Diameter

If a fluid flows in a pipe of constant diameter, but changing height, there is also work done on the fluid against the force of gravity.



Equating the work done with the change in potential energy gives:

$$P_1 + \rho g y_1 = P_2 + \rho g y_2$$

## General Bernoulli's Equation

The general case, where both height and speed may change, is described by Bernoulli's equation:

### Bernoulli's Equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

This equation is essentially a statement of conservation of energy in a (frictionless) fluid.

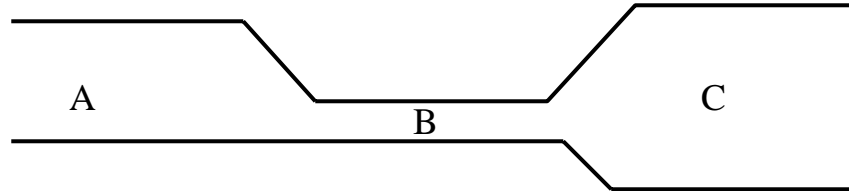


## Water Pipe

Water is flowing continuously in the pipe shown below. The velocity of the water is greatest at B.

$P + (1/2)\rho v^2 = \text{constant}$ . Where is the pressure in the water greatest?

- A)                      B)                      C)                      D) equal everywhere



## Example: Breathing

When a person inhales, air moves down the bronchus (windpipe) at 0.15 m/s. The average flow speed of the air doubles through a constriction in the bronchus. Assuming incompressible and frictionless flow, determine the pressure drop in the constriction.

**We apply Bernoulli's Eqn. and neglect the small change in vertical position.**

$$\text{obtain } P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) = \frac{1}{2}\rho[(2v_1)^2 - v_1^2] = \frac{3}{2}\rho v_1^2, \text{ or}$$

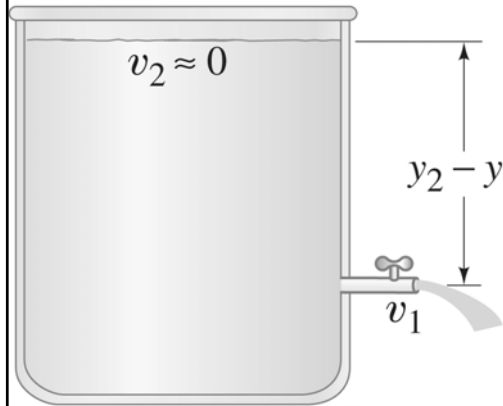
$$\Delta P = \frac{3}{2}(1.29 \text{ kg/m}^3)(15 \times 10^{-2} \text{ m/s})^2 = \boxed{4.4 \times 10^{-2} \text{ Pa}}$$

## Applications of Bernoulli's Principle:

Using Bernoulli's principle, we can find the speed of fluid coming from a spigot on an open tank:

Take  $P_2 = P_1 = P_{at}$ ; the Bernoulli Eqn becomes

$$\frac{1}{2}\rho v_1^2 + \rho g y_1 = \frac{1}{2}\rho v_2^2 + \rho g y_2$$



$$v_1^2 = 2gy_2 - 2gy_1$$

$$v_1 = \sqrt{2g(y_2 - y_1)}$$

0

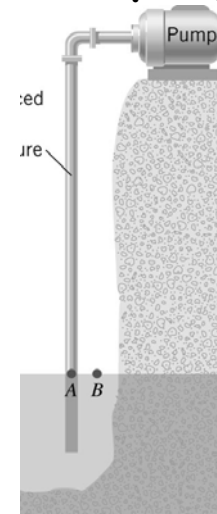
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## How high can you suck water up using a straw? (or pump water uphill)

In order to draw water (or any fluid) upward you must lower the pressure difference between the pressure inside the "straw" and the outside environment (usually the atmosphere).

1. Is there a limit to how low you can make the pressure inside a straw?
2. How high can you suck water?



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## Pumping Up Water

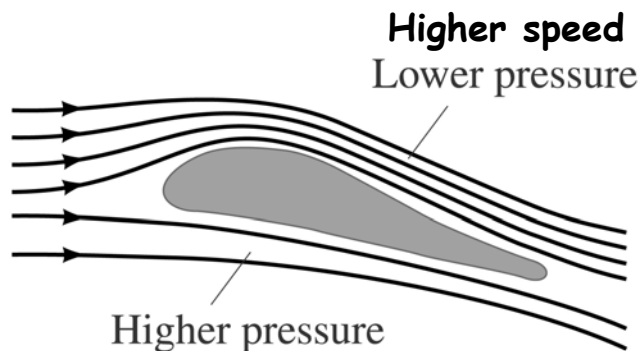
- $P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$
- Take  $v_1 = v_2 \approx 0$
- Take  $y_1$  (top of water) = 0
- $P_1 = P_{at}$  (top of water);  $P_2 = 0$
- $P_{at} = \rho g y_2$
- $y_2 = P_{at}/(\rho g) = 10^5 \text{ Pa}/[(10^3 \text{ kg/m}^3)g]$
- $y_2 \approx 10\text{m}$

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## Applications of Bernoulli's Principle: Airplane

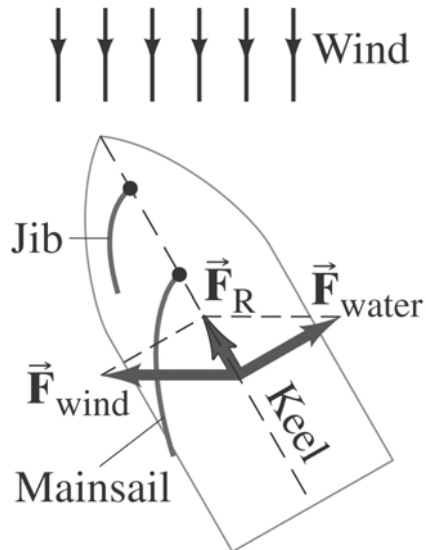
Lift on an airplane wing is due to the different air speeds and thus pressures on the two surfaces of the wing.



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## Applications of Bernoulli's Principle: Sailboat



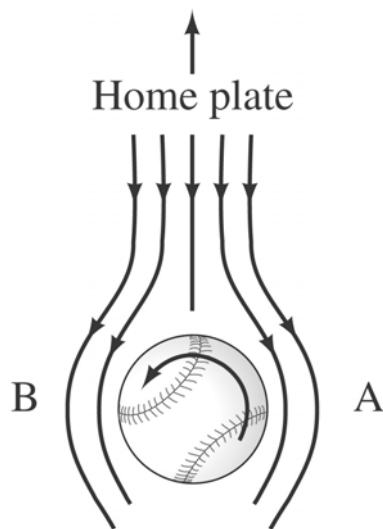
A sailboat can move against the wind, using the pressure differences on each side of the sail, and using the keel to keep from going sideways.

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## Applications of Bernoulli's Principle: Baseball



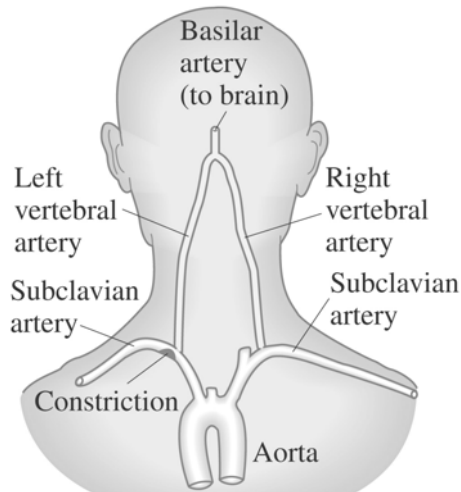
A ball's path will curve due to its spin, which results in the air speeds on the two sides of the ball not being equal.

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## Applications of Bernoulli's Principle: TIA



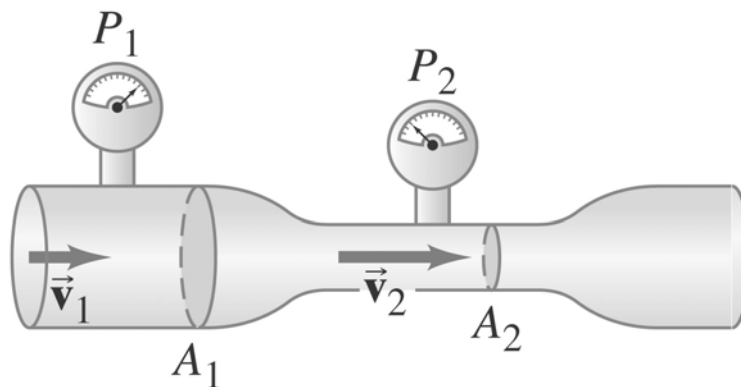
A person with constricted arteries will find that they may experience a temporary lack of blood to the brain (TIA) as blood speeds up to get past the constriction, thereby reducing the pressure.

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## Applications of Bernoulli's Principle

A venturi meter can be used to measure fluid flow speed by measuring pressure differences.



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## Fluid Dynamics Summary

- Equation of continuity for liquids:

$$A_1 v_1 = A_2 v_2$$

- Bernoulli's Eqn. for non-viscous fluids:

### **Bernoulli's Equation**

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

## End of Lecture 30

- For Friday, April 17, read Walker 15.8-9.
- Homework Assignment 15c is due at 11:00 PM on Friday, April 17.
- Midterm Review - Friday, April 17, 3:10 p.m., TH230