Surface defect gap solitons

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Abstract: We report on the existence of surface defect gap solitons. Such new type of solitons can be well supported by an interface between the defect of optical lattice and the uniform media with focusing saturable nonlinearity. The surface defect of optical lattice can profoundly affect the properties of solitons. It is shown that for the positive defect, stable solitons exist at the first bandgap and their powers decrease with defect depth; while for negative defect, stable solitons exist at the second bandgap and their powers increase with defect depth. Such solitons with moderate power between lower and higher ones cannot stably existent at the first bandgap.

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References and links

Defect modes (especially, defect solitons) have the potential applications for the all-optical switches and routing of optical signals. Defect modes in optical lattices have been reported [25-27]. Recently, linear defect modes in both one- and two-dimensional photonic lattices have been successfully observed [22, 23]. In a waveguide array with defect, the unstable in attractive defects but can remain stable in repulsive defects [21]. Recently, optical bandgaps but unstable in higher bandgaps; while at higher powers defect solitons become nonlinearity [20]. In particular, the defect of one-dimensional optical lattice features unique properties of solitons [21], for example, low-power defect solitons are linearly stable in lower bandgaps but unstable in higher bandgaps; while at higher powers defect solitons become unstable in attractive defects but can remain stable in repulsive defects [21]. Recently, optical lattices have been successfully generated [22, 23]. In a waveguide array with defect, the propagation of light pulses with a small inputting angle was investigated [24]. Linear defect modes in optical lattices have been reported [25-27]. Recently, linear defect modes in both one- and two-dimensional photonic lattices have been successfully observed [28]. Defect modes have been extensively studied in other systems e.g., in photonic crystals [29-34]. Defect modes (especially, defect solitons) have the potential applications for the all-optical switches and routing of optical signals.

1. Introduction
Surface waves are a type of waves which are formed at the interface between two media with different refractive index. Surface waves have been observed at interfaces of linear and photorefractive media [1], and at the interface with linear layered media [2]. Gap solitons may also exist at periodically modulated surfaces [3]. Surface solitons at the interface of uniform media and periodic waveguide arrays have been extensively investigated [4-11]. Specially, surface solitons were observed in experiments [12-15]. Nonlinear surface waves have been also studied in solid-state physics, nonlinear optics, and near-field optics [16-19].

Defect solitons are the nonlinear defect modes that bifurcate out from the linear defect modes. Such solitons have been investigated in the waveguide arrays with defocusing cubic nonlinearity [20]. In particular, the defect of one-dimensional optical lattice features unique properties of solitons [21], for example, low-power defect solitons are linearly stable in lower bandgaps but unstable in higher bandgaps; while at higher powers defect solitons become unstable in attractive defects but can remain stable in repulsive defects [21]. Recently, optical lattices have been successfully generated [22, 23]. In a waveguide array with defect, the propagation of light pulses with a small inputting angle was investigated [24]. Linear defect modes in optical lattices have been reported [25-27]. Recently, linear defect modes in both one- and two-dimensional photonic lattices have been successfully observed [28]. Defect modes have been extensively studied in other systems e.g., in photonic crystals [29-34]. Defect modes (especially, defect solitons) have the potential applications for the all-optical switches and routing of optical signals.
In all previous studies, however, the study of nonlinear defect modes in the form of surface wave has not been found yet. Therefore, the interesting issue is whether there exist surface nonlinear defect modes and their new properties at the interface between two media with different refractive index.

In this paper, we report on, for the first time to our knowledge, the existence of surface defect gap solitons (SDGSs). It is shown that stable SDGSs can be formed at the interface between the defect of optical lattice and uniform media with focusing saturable nonlinearity. The surface defect of lattice can provide new properties of solitons. For positive defect, stable SDGSs exist in the first bandgap and their powers decrease with the defect depth. For negative defect, stable SDGSs exist in the second bandgap and their powers increase with the defect depth. Their stabilities are also investigated.

2. The model

We consider beam propagation at the interface of uniform and one-dimensional optical lattice with a single-site defect in the focusing saturable nonlinear media described by the nonlinear Schrödinger equation. The nondimensionalized equation for the light field $q$ is [21]

$$i \frac{\partial q}{\partial z} + q_{xx} - \frac{E_0}{1 + I_L(x) + |q|^2} q = 0 ,$$

where $I_L$ is the intensity profile of the optical lattice when $x \geq \pi/2$,

$$I_L = I_0 \cos^2(x/\Omega)[1 + \varepsilon g(x)].$$

In Eq. (1), the diffraction length is $L_d=2k_1D^2/\pi^2$, $x$ is the transverse scale whose real unit is $D/\pi$, $E_0$ is the applied dc field whose real unit is $\pi^2/(k_0^2n_e^4D\gamma_{33})$, $I_0$ is the peak intensity uniform optical lattice, $\Omega$ describes lattice period (in this paper we take $\Omega=1$), $g(x)$ is a modulation function describing the shape of the defect, $\varepsilon$ is the defect depth, $D$ is the lattice spacing, $k_0=2\pi/\lambda_0$ is the wave number ($\lambda_0$ is the wavelength in vacuum), $k_l=k_0n_e$, $n_e$ is the unperturbed refractive index, and $\gamma_{33}$ is the electro-optic coefficient of the crystal. Here we assume that the defect is only a single lattice whose center is at $x=0$. We choose function $g(x)$ as $g(x)=\exp(-x^2/128)$. For positive defect $\varepsilon>0$, the light intensity $I_L$ of defect is higher than that without defect. While for negative defect $\varepsilon<0$, the light intensity $I_L$ of defect is lower than that without defect. For $\varepsilon=0$, optical lattices are uniform. The intensity distributions of defect lattice are shown for positive defect, negative defect and without defect in Figs. 1(a)-(c), respectively. We take typical parameters as $D=20\, \mu m$, $\lambda_0=0.5\, \mu m$, $n_e=2.3$, and $\gamma_{33}=280\, \text{pm/V}$, and take $I_0=3$ and $E_0=6$, which are typical in experimental conditions as shown in Ref. [21].

To show the existent conditions for SDGSs, we search the Floquet-Bloch spectrum by substituting a solution $q(x,z)=f(x)\exp(ikx+i\mu z)$ to the linear version of Eq. (1), where $\mu$ is the propagation constant, $k$ is the Bloch wave number and $f(x)$ is a periodic function [here $f(x)=f(x+\pi)$]. Figs. 2(a) and (b) show bandgap structure of the uniform lattice at $I_0=3$ and $E_0=6$. We now consider the interface between the defect of the optical lattice and the uniform media.
We search for the stationary soliton profiles in the form of \( q(x,z) = f(x) \exp(i\mu z) \), where \( f(x) \) is the real function satisfying the following equation

\[
\frac{\partial^2 f}{\partial x^2} - \frac{E_0}{1 + I_0(x) + |f|^2} f - \mu f = 0. 
\]  

(3)

The power \( P \) of a soliton is defined as \( P = \int_{-\infty}^{\infty} f^2(x) dx \). By numerically solving Eq. (3) using the shooting method, we get the soliton profiles. The soliton families are defined by propagation constant \( \mu \), lattice period \( \Omega \), defect depth \( \varepsilon \) and lattice modulation depth \( I_0 \). To indicate the stability of SDGSs, we search for the perturbed solution of Eq. (1) in the form \( q(x,z) = [f(x) + h(x,z) + ie(x,z)] \exp(i\mu z) \), where \( h, e \) are the real and imaginary parts of perturbation with a complex growth rate \( \delta \) upon propagation. By linearizing Eq. (1) around \( f(x) \) and keeping only the first-order terms in \( h(x,z) \) and \( e(x,z) \), one gets the eigenvalue problem:

\[
\partial h = -1/2 \partial^2 h \partial x^2 + \mu e + E_0(1 + I + f^2) \text{ and } \partial e = 1/2 \partial^2 e \partial x^2 - \mu h + E_0[1/(1 + I + f^2) - 2f^2/(1 + I + f^2)]^2. 
\]

We numerically solve these coupled equations to get the growth rate \( Re(\delta) \).

The ability of periodic structures to dramatically modify beam diffraction leads the surface modes to be supported at or near the interface. The interface properties (e.g., the lattice depth and period) significantly affect soliton properties. In addition, gap solitons are formed when light waves experience Bragg scattering from the periodic structure. Therefore, we predict that by modulating the depth of the single lattice in the interface, some new properties of surface modes could be produced.

3. Numerical results

To further study the SDGSs’ robustness, in all numerical simulations based on Eq. (1), we add a noise to the inputted SDGSs by multiplying them with \([1+\rho_1,2(x)]\), where \( \rho_1,2(x) \) is a Gaussian random function with \( \langle \rho_{1,2}^1 \rangle = 0 \) and \( \langle \rho_{1,2}^2 \rangle = \sigma_1,2 \) (we choose that \( \sigma_1,2 \) is equal to 10% the input soliton amplitude). For \( \varepsilon > 0 \), we find that SDGSs are only at the first bandgap; while for \( \varepsilon < 0 \), SDGSs are only at the second bandgap, which is similar to the report in Ref. [21]. Figures 2(a) and 2(c) show that the power of SDGSs depends on the eigenvalue for \( \varepsilon = 0.5 \) and \( \varepsilon = -0.5 \), respectively. In addition, we find that the power is monotonically increasing for \( \varepsilon = 0.5 \) in the domains of \(-0.9 \leq \mu \leq 0.8 \) (-0.9 corresponds to A) and \( \mu \leq 1.8 \) (-1.8 corresponds to C) [Fig. 2(a)] and for \( \varepsilon = -0.5 \) in the domains of \(-3.64 < \mu < -2.95 \) [Fig. 2(c)], i.e., \( dP/d\mu > 0 \), which implies in these domains stable SDGSs are formed according to the Vakhitov-Kolokolov criterion. However, the power of SDGS is not monotonically increasing in the domain of eigenvalue \(-1.8 < \mu < 0.9 \) for \( \varepsilon = 0.5 \) [Fig. 2(a)], which implies SDGSs can not stably propagate in this domain. We also calculate the growth rate of perturbation by solving the coupled equations in section 2. The growth rate of perturbation for \( \varepsilon = 0.5 \) shown in Fig. 2(b) indicates the instability of solitons in the domain of eigenvalue \(-1.8 < \mu < 0.9 \), which demonstrates further the above analytic result. We perform numerical simulations by selecting the parameters of A, B and D in Fig. 2(a) and E, F and G in Fig. 2(c). The results, plotted in Figs. 2(d), 2(e), 2(g), 2(h), and 2(i)-2(o), show that SDGSs can stably propagate for a distance of \( z = 200 \). Figures 2(f) and 2(i), corresponding to B in Fig. 2(a), show that soliton with \( \mu = 1.4 \) is unstable upon propagation. All these simulations prove the above analytic results. These results show: (1) both lower- and higher-power SDGSs can stably exist at the first bandgap (corresponding to the positive defect) and the second bandgap (corresponding to the negative defect), which is different from the results in Ref. [21] where low-power defect solitons are linearly stable at the first bandgap and at higher powers defect solitons become unstable in positive defects, but can be stable in negative defects; (2) the stable SDGSs with moderate power between the lower and higher ones cannot exist at the first bandgap but exist at the second bandgap, which is different from the results in Ref. [21] where at the first bandgap, the power increases solitons alternate between stable and unstable regions as the power goes up.
Next we analyze how the defect depth affects soliton profiles for a fixed eigenvalue $\mu$. For $\mu$=-2.1, SDGSs exist in the range of defect depth 0< $\varepsilon$ <0.75. Figs. 3(a)-(c) display three SDGS profiles at $\varepsilon$=0.1, $\varepsilon$=0.5, and $\varepsilon$=0.7, and their stable propagations are shown in Figs. 3(d)-(f), respectively. When $\mu$=-3.2, stable SDGSs exist in the range of defect depth -0.95 < $\varepsilon$ < -0.25. Figs. 3(g)-(i) show three SDGS profiles for the defect formed at $\varepsilon$=-0.3, $\varepsilon$=-0.5, and $\varepsilon$=-0.9, and their stable propagations are shown in Figs. 3(j)-(l), respectively. Figures 4(a) and 4(b) show the stable/unstable regions of SDGSs at the first and the second bandgaps, respectively. The power of SDGS is linearly decreased and increased with the increase of defect depth for positive and negative defects, respectively, shown in Figs. 4(c) and 4(d).
Finally, we give the surface gap solitons at the interface between uniform media and the optical lattice, which is actually a particular case of defect $\varepsilon = 0$. The surface gap solitons with $\varepsilon = 0$ have been investigated in another system with Kerr-type media [6]. We find that surface gap solitons also exist in the system with focusing saturable nonlinearity modeled by Eq. (1). Figure 5(a) show the power of surface gap solitons versus eigenvalue. Figures 5(b) and 5(e) show that surface gap solitons exhibit stable propagation for $\mu = -2.4$ and $\mu = -2.05$, respectively, whose power decreases monotonically from point A ($\mu = -2.05$) to B ($\mu = -2.4$).

4. Summary

To summarize, we reveal the existence of surface defect gap soliton. Such new type of solitons are supported by an interface between the defect of optical lattice and the uniform media with focusing saturable nonlinearity when light wave at the interface experiences Bragg scattering. The surface defect of optical lattice offers new properties of solitons. Their powers decrease and increase with the defect depth for the positive defect and the negative defect, respectively. Such solitons with lower and higher power can exhibit stable propagation at the first and second bandgaps, while those with moderate power can stably exist at the second bandgap but cannot stably exist at the first bandgap.

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