Reshaping the trajectory and spectrum of nonlinear Airy beams

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We demonstrate theoretically and experimentally that a finite Airy beam changes its trajectory while maintaining its acceleration in nonlinear photorefractive media. During this process, the spatial spectrum reshapes dramatically, leading to negative (or positive) spectral defects on the initial spectral distribution under a self-focusing (or defocusing) nonlinearity. (© 2012 Optical Society of America)

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1 Since the first prediction and experimental demonstration of optical Airy beams [1,2], many efforts have been made to maintain or control their self-accelerating trajectories in both linear [3–7] and nonlinear [8–12] media. Unfortunately, nonlinearities tend to destroy the phase of Airy beams and thus to break down the acceleration, especially under the action of a self-focusing nonlinearity [10,11]. Formation of accelerating self-trapped optical beams has been proposed employing different self-focusing nonlinearities [12], and quite recently, nonlinear accelerating Airy beams were explored experimentally with Kerr and quadratic nonlinear media [13,14].

In this Letter we propose a scheme to control both the trajectories and the spatial spectra of nonlinear Airy beams. By altering the initial conditions through laterally shifting the cubic phase mask coded on a spatial light modulator, we demonstrate experimentally and theoretically that a finite Airy beam can change its trajectory while maintaining its acceleration in a nonlinear photorefractive medium. Furthermore, the spatial spectrum reshapes dramatically during nonlinear propagation leading to positive (or negative) spectral defects under a self-focusing (defocusing) nonlinearity. Such peculiar nonlinear phenomena may find applications in beam/pulse shaping as well as in laser filamentation and plasma guidance.

Let us start our analysis with the one-dimensional (1D) normalized, nonlinear equation under the paraxial and slow varying amplitude approximations, applicable for beam propagation under a saturable nonlinearity [10,12]:

\[
\partial \Psi / \partial \xi = 0.5i \alpha^2 \Psi / \partial s^2 - i \gamma \Psi / (1 + |\Psi|^2),
\]

where \(s\) and \(\xi\) indicate normalized transverse and longitudinal coordinates, respectively equal to \(x / x_0\) and \(z / (k_0 n_0 a_0^2)\), in which \(x_0\) is an arbitrary length scale, \(k_0\) is the vacuum wave number and \(n_0\) is the linear refractive index of the photorefractive crystal. In Eq. (1), \(\Psi\) is the complex amplitude of the optical field, and \(\gamma = -k_0^2 n_0 a_0^2 r_{33} E_0^2 / 2\) is the normalized nonlinear coefficient with \(r_{33}\) being the electro-optics coefficient for the extraordinarily polarized beams and \(E_0\) being the bias field. \(\gamma > 0\) \((< 0)\) represents a self-focusing (defocusing) nonlinearity, respectively.

In the linear regime \([i.e., \gamma = 0]\), the evolution of an Airy beam can be altered at ease through shifting the cubic phase mask in the Fourier space [4].

\[
\phi(s, \xi) = \sqrt{I_0} f(s, \xi) \exp(-i \omega_m^2 x^2 + i 2 \alpha \omega_m x) \times A|x - \omega_m (\xi/2)^2 + i \alpha (\xi + 2 \omega_m)| \exp(i \omega_m s), \quad (2a)
\]

\[
f(s, \xi) = \exp[as + is \xi^2/2 - (i \omega_n^2 s + i \alpha^2 x^2 - 2i \omega_m)] \times \exp[-i \alpha^2 / 2i \omega_m x^2], \quad (2b)
\]

where \(I_0\) is the peak intensity, \(a\) is the truncation factor, and \(\omega_m\) is the shift of the cubic phase mask. From Eq. (2), it can be seen that the Airy beam accelerates following the parabolic trajectory \(s = \omega_m (\xi^2/2)\), and the peak intensity \((I_0)\) appears at \(\xi = -2 \omega_m^2\). In Fig. 1(a1–a3), three examples of numerically generated Airy beams are shown corresponding to launching the input \(\phi(s, \xi = 0)\) at \(\omega_m = 0, -1, \) and \(-6\). As expected, the peak intensity appears at \(\xi = 0, 2,\) and 12, respectively. When the nonlinearity \((|\gamma| = 2.72\) is used in the whole paper) is turned on, Airy beams \((I_0 = 4.08\) and \(a = 0.08)\) with different \(\omega_m\) behave drastically different. Under a self-focusing nonlinearity \((\gamma > 0)\), the evolutions of the Airy beams at \(\omega_m = 0, -1, \) and \(-6\) are shown in Fig. 1(b1–b3). In the case of \(\omega_m = 0\), most of the Airy beam turns into an “off-shooting” soliton [Fig. 1(b1)]. However, at \(\omega_m = -6\), the Airy profile and accelerating properties are preserved [Fig. 1(b3)], while overall the transverse beam width shrinks when compared to that associated with linear propagation depicted in Fig. 1(a3). If the absolute \(\omega_m\) is not high enough, e.g., \(\omega_m = -1\), only part of the Airy beam profile survives, whereas the other part undergoes “off-shooting” along tangential directions [Fig. 1(b2)]. Detailed simulation results indicate that when \(\omega_m = 0\), even a weak self-focusing nonlinearity would affect the trajectory of the Airy beam [13,14]. However, the scheme...
described in Fig. 1(b3) seems to be an efficient way to maintain the acceleration property of the nonlinear Airy beam. On the other hand, Airy beams perform much better under a self-defocusing nonlinearity (γ < 0), as the acceleration persists for all cases and even under the condition α_{in} = 0, although in these cases the transverse beam width broadens slightly [Fig. 1(c1–c3)].

To have a better understanding of the nonlinear Airy beam at α_{in} = –6, let us look into the corresponding spectra. Although the beam dynamics look similar in the linear and nonlinear cases except for the lateral shifts (towards the positive and negative s direction under a self-focusing and -defocusing nonlinearity, respectively), their spectral distributions have dramatic differences, as shown in Fig. 2. The spectrum of an exponentially truncated Airy beam in the linear regime has a Gaussian shape without any change along the propagation direction [Fig. 2(a)]. However, in the presence of a self-focusing nonlinearity, a distance-dependent self-shifting spectral gap appears from ξ = 7 to ξ = 17 (regions where the Airy beam intensity is high), as shown in Fig. 2(b).

After this spatial-spectral gap emerges, most of the power spectra concentrate in the vicinity of the gap with surrounding ripples. However, before ξ = 7 and after ξ = 17, the spectrum keeps the Gaussian distribution since the nonlinearity has no, or only a weak, effect on the Airy beam in low-intensity regions. Likewise, under a self-defocusing nonlinearity, the spectrum of the Airy beam exhibits a positive distance-dependent self-shifting defect [Fig. 2(c)]. This nonlinear spectrum reshaping phenomenon is robust even in presence of high-level noise (results not shown here), providing an experimentally practical yet effective way for spectrum selection, and the idea could be readily extended to the temporal spectral domain.

Based on recent work in [12–14], one may postulate that the persistence of the acceleration of finite-energy Airy beams might originate from the existence of ideal infinite-energy nonlinear self-accelerating modes. To find those nonlinear modes, let us define ζ = s − ℏζ^2/2 (ℏ is the rate of acceleration), in such a way that Eq. (1) becomes

\[ \partial^2 \Psi/\partial \xi^2 - \hbar \xi \partial \Psi/\partial \xi = 0.5 \hbar \partial^2 \Psi/\partial s^2 - i \hbar \Psi/(1 + |\Psi|^2). \]  (3)

By inserting Ψ = u(ζ) exp(iℏζ^2/6 + iℏζξ) into Eq. (3) and letting u be a real function, we get

\[ d^2 u/dζ^2 - 2\hbar \xi u - 2u/(1 + u^2) = 0. \]  (4)

By solving Eq. (4) numerically (here we use ℏ = 1/2), solutions with different peak intensities are found. Typical results are shown in Fig. 3, where self-focusing and defocusing nonlinearities lead to slightly shrinking and broadening of the main Airy lobe with respect to the linear infinite Airy beam, in agreement with the beam propagation simulation presented in Fig. 1. The spectra of Figs. 3(a) and 3(d) are plotted in Figs. 3(b) and 3(e), where the formation of negative and positive spectral defects is clearly evident. Moreover, the spectrum in Fig. 3(b) has more noticeable ripples outside the defect compared to the one in Fig. 3(e). After Fourier transforming Ψ = u(ζ)

![Figure 1](image1.png)  
**Fig. 1.** (Color online) Linear (row 1) and nonlinear (rows 2 and 3) propagation of an Airy beam under different conditions. The panels from left to right correspond to different shifting α_{in} = 0, −1, and −6 of the cubic phase mask, and from top to bottom to beam propagation under (a) linear, (b) nonlinear self-focusing, and (c) nonlinear self-defocusing conditions.

![Figure 2](image2.png)  
**Fig. 2.** (Color online) Spatial power spectra of Airy beams at α_{in} = −6 during beam propagation under (a) linear, (b) nonlinear self-focusing, and (c) nonlinear self-defocusing conditions.

![Figure 3](image3.png)  
**Fig. 3.** (Color online) Nonlinear self-accelerating solutions under saturable nonlinearities. (a) and (d): Self-accelerating modes under self-focusing and -defocusing nonlinearities, respectively. (The red dashed line corresponds to a linear infinite Airy beam); (b) and (e): Spectra corresponding to (a) and (d); (c) and (f): spectral distribution during the nonlinear propagation of the self-accelerating modes in (a) and (d), respectively.
With regard to this aspect, the spectral defects in Figs. 4(a–c) and 4(d–f) correspond, respectively, to the cases in which the peak intensity is set at the middle (a–c) and output (d–f) of the crystal. Figs. 4(b–f) show the averaged spectral distributions of the outputs in (a1–f1). The downward pointing (red) arrows indicate the position of the spectral defects in k-space.

\[
\text{exp}(i h^2 \xi^2 / 6 + ih \xi \tilde{\xi}), \text{we can get } \Phi(\omega, \xi) = \exp[-i h^2 \xi^2 / 6] U(\omega - h \xi), \text{where } \Phi = F[\Psi(s, \xi)], \text{and } U = F[u(s)].
\]

The above predicted phenomena have been observed in our experiment, carried out in a biased photorefractive crystal with a setup similar to that used in [10], except that here we employ a 1D cubic phase mask and a cylindrical lens for 1D Airy beam generation. In our experiments, the absolute value of the bias voltage is kept at 600 V across the 5 mm-wide crystal (corresponding to \(|\gamma| = 2.72\) as used in our simulation). By shifting the cubic mask, we conveniently set the peak intensity of the Airy beam in the middle [as indicated by line II in Fig. 1(a3)] of the crystal along the propagation direction \(\xi\) [4]. Figure 4(a1) shows the linear output profile whose main hump has the same position of the input and locates at the right side of the peak intensity. The dashed lines I and III in Fig. 1(a3) mark the input and output facets of the crystal, respectively, which in turn indicate the nonlinear regime along the propagation direction in the simulation (from \(\xi = 7\) to 17, as the propagation outside the crystal is basically linear). The dashed vertical line in Fig. 1(a3) also marks the transversal positions of the peak intensity for the linear beam, as does the one in Fig. 4(a1). As mentioned before, the evolution is almost linear at \(\xi < 7\), so the nonlinear evolution in the experiments is similar to the simulation shown in Figs. 1(a3–c3). Under a self-focusing nonlinearity, the main hump of the output is not self-trapped, but keeps its acceleration [Fig. 4(b1)]. In addition, it shrinks and shifts to the right when compared to the linear case.

On the other hand, under a self-defocusing nonlinearity, the main hump expands and shifts to the left when compared to the linear output [Fig. 4(c1)]. Meanwhile, negative and positive defects appear in the initial Gaussian spectrum [presented in Fig. 4(a2)] under self-focusing and -defocusing nonlinearities, respectively. Those defects move to the center of the Fourier space [Figs. 4(b2) and 4(c2)]. By shifting the crystal along the \(\xi\)-direction, it is possible to position the output facet at the dashed line II in Fig. 1(a1). The beam evolutions [Figs. 4(d3–f3)] have a similar behavior to those shown in Figs. 4(a3–c3) and Figs. 2(a–c).

In summary, we have demonstrated nonlinear control of both trajectories and spatial spectra of accelerating Airy beams. Such nonlinear control may be applicable to nonparaxial accelerating beams studied recently [15–17], and may prove useful for exploiting these beams for various important applications.

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References

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