Elimination of transverse instability in stripe solitons by one-dimensional lattices

Jianke Yang,[1,*] Daniel Gallardo,[2] Alexandra Miller,[2] and Zhigang Chen[2]

[1]*Department of Mathematics and Statistics, University of Vermont, Burlington, Vermont 05401, USA*

[2]*Department of Physics and Astronomy, San Francisco State University, San Francisco, California 94132, USA*

**Corresponding author: jyang@math.uvm.edu*

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We demonstrate theoretically and experimentally that the transverse instability of coherent soliton stripes can be greatly suppressed or totally eliminated when the soliton stripes propagate in a one-dimensional photonic lattice under self-defocusing nonlinearity.

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It is well known that in homogeneous nonlinear optical media, a bright soliton stripe, uniform along the transverse stripe (say, y) direction but localized along the orthogonal transverse (say, x) direction, is unstable upon propagation along the longitudinal z direction when transverse perturbations are present [1–9]. When a one-dimensional (1D) optical lattice is introduced along the x or y direction, the soliton stripe is still transversely unstable under self-focusing nonlinearity [10,11]. To suppress this transverse instability, some ideas have been proposed. For instance, this instability can be completely eliminated if the soliton stripe is made sufficiently incoherent along the transverse direction [12]. This instability can also be significantly reduced by nonlinearity saturation or incoherent mode coupling [13,14]. The introduction of two-dimensional (2D) square lattices can also suppress transverse instability, but the transversely stable structures in such media are soliton trains (comprising an infinite array of intensity peaks) rather than soliton stripes [15,16].

In this Letter we demonstrate, both theoretically and experimentally, that transverse instability of coherent soliton stripes is greatly suppressed or totally eliminated when the soliton stripes propagate in a 1D lattice under self-defocusing nonlinearity.

Our theoretical model is the 2D NLS equation with a 1D lattice:

$$iU_z + U_{xx} + U_{yy} + n(x)U + \sigma|U|^2U = 0, \qquad (1)$$

where $\sigma = \pm 1$ denotes self-focusing and self-defocusing nonlinearity, and $n(x)$ is a 1D lattice. For definiteness, we take

$$n(x) = -6\sin^2 x$$

in this Letter [see Fig. 1(a)]. Stripe (1D) solitons in this model are of the form $U(x,y,z) = u(x)e^{-i\mu z}$, where $u(x)$ is a real-valued localized function and μ is the propagation constant. When $\sigma = -1$ (defocusing nonlinearity), there is a stripe-soliton family in the first bandgap of the lattice. The power curve of this family is shown in Fig. 1(b). Here, the power P is defined as $\int_{-\infty}^{\infty}|u|^2 \mathrm{d}x$. The intensity profile of the soliton at $\mu = 4.5$ is displayed in Fig. 1(c) (the peak intensity is approximately 3.2). To determine the transverse stability of these stripe solitons, we perturb them as

$$U(x,y,z) = e^{-i\mu z}\{u(x) + [v(x) + w(x)]e^{iky+\lambda z} + [v^*(x) - w^*(x)]e^{-iky+\lambda^* z}\},$$

where $v, w \ll 1$ are normal-mode perturbations and k is the transverse wavenumber. Substituting this perturbed solution into Eq. (1) and neglecting higher order terms of (v, w), we obtain the linear-stability eigenvalue problem

$$L_0 w = -i\lambda v, \qquad L_1 v = -i\lambda w,$$

where

$$L_0 = \partial_{xx} + n(x) + \mu - k^2 + \sigma u^2,$$
$$L_1 = \partial_{xx} + n(x) + \mu - k^2 + 3\sigma u^2,$$

and λ is the eigenvalue. The full spectrum of this eigenvalue problem can be obtained numerically by the Fourier collocation method [9]. For the stripe soliton at

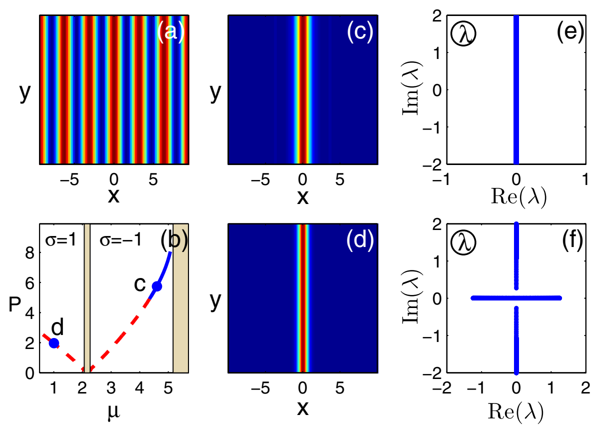

Fig. 1. (Color online) (a) 1D lattice, (b) power curves of stripe solitons (dashed red indicates instability, solid blue indicates stability, and shaded regions are Bloch bands), (c) intensity profile of a stripe soliton in the first gap (at $\mu = 4.5$) under defocusing nonlinearity, (d) intensity profile of a stripe soliton in the semi-infinite gap (at $\mu = 1$) under focusing nonlinearity, (e), (f) stability spectra of stripe solitons in (c), (d), respectively.

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$\mu = 4.5$ [see Fig. 1(c)], its stability spectrum is displayed in Fig. 1(e). This spectrum lies entirely on the imaginary axis, indicating that this stripe soliton is transversely stable under defocusing nonlinearity! In contrast, when the nonlinearity is self-focusing ($\sigma = 1$), stripe solitons will remain transversely unstable and this instability is strong. To demonstrate, a family of stripe solitons in the semi-infinite gap under focusing nonlinearity is obtained, and its power curve is plotted in Fig. 1(b). At $\mu = 1$ of the power curve, the soliton is shown in Fig. 1(d) (its peak intensity is roughly 1.7). The linear-stability spectrum of this soliton is displayed in Fig. 1(f). This spectrum contains large positive eigenvalues (with the maximum 1.25), indicating strong instability.

Next we consider the linear stability of other stripe solitons in the solution families of Fig. 1(b). When the solitons are near an edge μ_0 of a Bloch band, these solitons under perturbations are low-amplitude Bloch-wave packets:

$$U(x, y, z) = e^{-i\mu_0 z}[\epsilon\Psi(X, Y, Z)p(x) + \epsilon^2 U_2 + \cdots],$$

where $p(x)$ is the Bloch wave at edge μ_0, $0 < \epsilon \ll 1$, $X = \epsilon x$, $Y = \epsilon y$, $Z = \epsilon^2 z$, and $\Psi(X, Y, Z)$ satisfies

$$i\Psi_Z + D\Psi_{XX} + \Psi_{YY} + \sigma\alpha|\Psi|^2\Psi = 0, \qquad (2)$$

with D being the diffraction coefficient of the 1D lattice at edge μ_0 and $\alpha > 0$ being a constant [9]. Stripe solitons in Eq. (1) correspond to stripe envelope solutions $\Psi(X, Y, Z) = A(X)e^{-i\tau Z}$, where $\mathrm{sgn}(\tau) = -\mathrm{sgn}(\sigma) = -\mathrm{sgn}(D)$, and $A(X)$ is a sech function [9]. It is well known that this stripe envelope solution is transversely unstable in the envelope equation (2) [1,7,9]. Thus, low-amplitude soliton stripes in Eq. (1) are all transversely unstable. In particular, for the stripe-soliton family in the semi-infinite gap, $D > 0$, thus the transverse instability is of the neck-type (due to positive eigenvalues); while for the soliton family in the first gap, $D < 0$, thus the transverse instability is of the snake type (due to both positive and complex eigenvalues) [9]. The magnitude of these unstable eigenvalues is proportional to $\sqrt{|\mu - \mu_0|}$ [16].

Away from band edges, we have tracked these transverse-instability eigenvalues for the two soliton families in Fig. 1(b). We find that for the soliton family in the semi-infinite gap (under focusing nonlinearity), as μ moves away (decreases) from the band edge $\mu_0 = 2.06$, the magnitude of the largest positive eigenvalue keeps increasing [at $\mu = 1$, this magnitude has reached 1.25; see Fig. 1(f)]. Thus, all stripe solitons in this family are transversely unstable. This instability is of the neck type and is strong when μ is not near the band edge.

However, for the soliton family in the first gap under defocusing nonlinearity, the story is very different. In this case, as μ moves away (increases) from the band edge $\mu_0 = 2.27$, the positive eigenvalues of the snake instability quickly disappear when $\mu > 2.3$. Meanwhile, the real parts of the complex eigenvalues in the snake instability first saturate quickly to a very low level when $2.4 < \mu < 4.3$. After $\mu > 4.3$, these complex eigenvalues then totally disappear; hence, stripe solitons in this μ range are all stable! To demonstrate, the most unstable

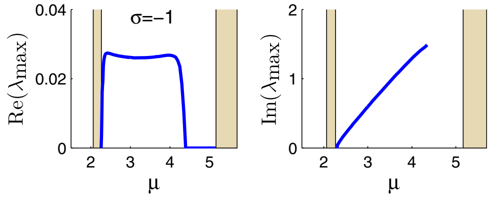

Fig. 2. (Color online) The most unstable eigenvalue $\lambda_{\max}$ versus μ for stripe solitons in the first gap under defocusing nonlinearity: (left) real part, (right) imaginary part.

eigenvalue $\lambda_{\max}$ versus μ is plotted in Fig. 2. It is seen that the real part of $\lambda_{\max}$ (the maximum growth rate) is under 0.03 for all stripe solitons, indicating that the transverse instability is extremely weak. In addition, when $\mu > 4.3$, the real part of $\lambda_{\max}$ becomes zero, hence those stripe solitons are linearly stable. These results show that under defocusing nonlinearity, transverse instability of stripe solitons is either greatly suppressed or totally eliminated.

The above linear-stability results are also corroborated by nonlinear-evolution simulations of these stripe solitons under random-noise perturbations. For the soliton in Fig. 1(c), its initial perturbed state (with 2% random-noise perturbations) is shown in Fig. 3(a), and evolution output of this perturbed soliton under defocusing nonlinearity at $z = 100$ is shown in Fig. 3(b). It is seen that even after such a long evolution, this stripe soliton still remains robust and does not break up. In contrast, when the soliton in Fig. 1(d) is perturbed by the same amount of perturbations, after evolution under focusing nonlinearity, this stripe quickly breaks up into filaments at $z = 3$ [see Fig. 3(c)]. Thus suppression of transverse instability under defocusing nonlinearity and persistence of transverse instability under focusing nonlinearity hold for nonlinear evolutions as well.

Experimentally, we have confirmed the above theoretical predictions. The experiments were performed in a 10 mm long biased SBN:60 photorefractive crystal. The optically induced 1D lattice (41 μm lattice spacing) is shown in the upper row, first column of Fig. 4, while the initial stripe beam (12 μm FWHM) is shown in the lower first column. The peak intensity ratio between the probe and lattice beams is about 1:5. In the presence of the lattice, this probe beam exhibits strong discrete diffraction during linear propagation (upper row, second column). It breaks up due to strong transverse instability under self-focusing nonlinearity at a bias field of 2 kV/cm (upper row, third column), but remains robust against

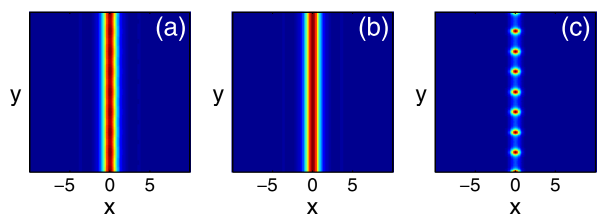

Fig. 3. (Color online) (a) Initial intensity pattern of the stripe soliton in Fig. 1(c) under 2% perturbations, (b) output intensity of the perturbed soliton in (a) after nonlinear evolution of $z = 100$ under defocusing nonlinearity, (c) output intensity of the perturbed soliton in Fig. 1(d) after nonlinear evolution of $z = 3$ under focusing nonlinearity.

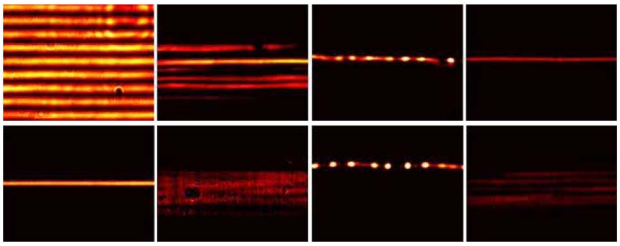

Fig. 4. (Color online) Experimental results. Upper row, first column, 1D lattice; lower row, first column, initial probe beam; upper row, results with lattice; lower row, results without lattice; second column, linear diffraction; third column, output with focusing nonlinearity; fourth column, output with defocusing nonlinearity. The bias field and crystalline c axis are along the vertical direction.

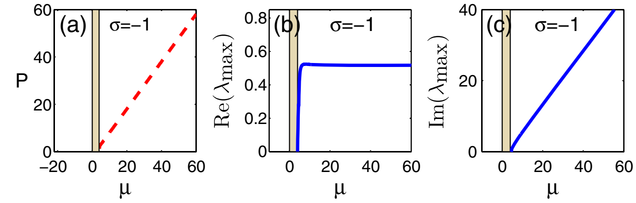

Fig. 5. (Color online) (a) Power curve of discrete stripe solitons under defocusing nonlinearity, (b), (c) real and imaginary parts of the most unstable eigenvalue $\lambda_{\max}$ versus μ.

transverse perturbations under defocusing nonlinearity at a bias field of −1.6 kV/cm (upper row, fourth column). As a comparison, when no lattice is induced inside the crystal, the corresponding results are shown in the lower panels of Fig. 4. In this case, the stripe beam breaks up strongly due to neck-type transverse instability under focusing nonlinearity [3], and broadens even more than linear diffraction under defocusing nonlinearity.

It is important to notice from Figs. 1(b) and 2 that transversely stable stripe solitons (under defocusing nonlinearity) are located near the second Bloch band, implying that their stability is caused by mode coupling between the first and second bands. This means that the existence of these stable stripe solitons cannot be predicted by the corresponding discrete NLS model

$$iU_{n,z} + U_{n+1} - 2U_n + U_{n-1} + U_{n,yy} + \sigma |U_n|^2 U_n = 0$$

since this discrete model is derived under a single-band approximation and it does not incorporate mode coupling between different Bloch bands [17]. To confirm this, we seek stripe solitons in this discrete model as $U_n(y,z) = u_n e^{-i\mu z}$, where μ is the propagation constant. The power curve of these solitons $P = \sum |u_n|^2$ versus μ for $\sigma = -1$ (defocusing nonlinearity) is plotted in Fig. 5(a), and the most unstable linear-stability eigenvalue of each soliton versus μ is plotted in Figs. 5(a)–5(c). It is seen that when μ moves away (increases) from the band edge $\mu_0 = 4$, the real part of the most unstable eigenvalue quickly saturates (to about 0.52) and then stays at this level for all higher μ, thus transverse instability persists for all these discrete stripe solitons under defocusing nonlinearity.

In summary we have demonstrated both theoretically and experimentally that the transverse instability of coherent soliton stripes is totally eliminated or greatly suppressed when the soliton stripes propagate in a 1D photonic lattice under self-defocusing nonlinearity. This elimination of transverse instability makes stripe solitons applicable in physical settings.

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