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Chapter 1

1.1 Non-Linear Optics

For centuries the interactions between light and matter have been understood to have a linear relationship. As light passes through an optical medium we can predict the properties of the outgoing beam by knowing the wavelength, $\lambda$, and the wave’s group velocity, $v$. In the last few decades, Starting with the advent of lasers, extensive experimental and theoretical studies have shown that at high enough intensities the relationship between light and matter becomes non-linear in nature, which turns to be the main subject of nonlinear optics.

In order to understand the non-linear properties of light it is important to have some background on the wave nature of light. Mathematically, a plane wave can be described as:

$$\vec{E} = E_o \exp(ikx - \omega t)$$

where $E_o$ is the Amplitude, $k$ is the wave number, $k = \frac{2\pi}{\lambda}$ and $\omega$ is the angular frequency. When this electric field propagates through material the resulting polarization of the material will determine how it will respond to light. This relationship is defined by:
\[ \vec{P} = \varepsilon_o \chi \vec{E} \]

where \( \vec{P} \) is the induced dipole moment, \( \varepsilon_o \) is the permittivity in a vacuum, \( \chi \) is the electric susceptibility of the material, and \( \vec{E} \) is the electric field imposed by the light. For low intensities this equation is sufficient in describing the physical properties of light passing through material. The polarization of the dipoles is parallel to and directly proportional to the oscillating incident light. At higher intensities the motion of the dipoles will become distorted and nonlinear terms will be important. Provided that these new terms are still small compared to the linear term we can expand the polarization in a Taylor expansion [1]:

\[ \vec{P} = \varepsilon_o \chi \vec{E} + \varepsilon_o (\chi_1 \vec{E}^2 + \chi_2 \vec{E}^3 + \text{higher order terms}) \]

As previously mentioned, the high intensity of light (i.e. large E field) will cause these higher order effects to become substantial. Alternatively, by controlling the properties of the material (i.e. increasing the higher order \( \chi \) terms), non-linear effects can be achieved with relatively low beam intensity.

For the purposes of our experiments only relatively low beam intensity was needed due to the unique property of the photorefractive materials, which have one of the most intriguing effect found in modern nonlinear optics: the photorefractive effect [2]. For the work
presented in this thesis, a photorefractive crystal made up of Strontium Barium Niobate 60 (SBN-60) was used. This crystal provides the desired photorefractive effect as explained below.
1.2 Photorefractive Effect

A photorefractive crystal is created by adding impurities to the material’s structure while it grows. When the crystal is being formed heat and compression in combination with a strong electric field are applied to the crystal along the crystalline c-axis. The result is a crystal with a strong electrooptic effect along this c-axis. As discussed above, this electrooptic effect translates into a dominating $\chi_2$ nonlinearity otherwise known as Pockel’s effect. The change in refractive index due to the Pockel’s effect is proportional to the electric field as:

$$\Delta n = -\frac{1}{2}(n_0 r_{\text{eff}} E)$$

where $r_{\text{eff}}$ is the electrooptical tensor coefficient, $n_0$ is the index of refraction with no electric field present, and $E$ is the electric field magnitude.

Dopants imbedded between the conduction and valence bands of the crystal are the contributors that make possible the electrooptic effect and therefore the change in the index of refraction. When light enters a photorefractive crystal, the dopants can become excited from the middle band gap through a process of photoionization [2]. Energy carried by small intensities of light can cause excited electrons to jump into the conduction band, leaving ionized donors behind. (See
Figure 1.1) Essentially, the electrons are free to move through the crystal until they find a potential well in the form of an acceptor. A uniform plane wave will excite the dopants when it enters the crystal, but electrons may be recombined before they can migrate and get trapped by the acceptors in the crystal. It is when the incoming light does not have a uniform intensity pattern that photorefractive materials give rise to interesting effects.

![Diagram of electronic bands](image)

Figure 1.1
Defects in the crystal are located within the band gap. A photon can cause the donor atom to ionize, allowing an electron to be excited into the conduction band to move freely through the crystal. Steady state occurs when the electrons recombine with an ionized donor. The acceptor atoms are fully occupied and are therefore inactive. [3]
Consider a non-uniform optical beam entering a photorefractive crystal with energy sufficient enough to excite the electrons in the donor level to the conduction band, but not enough to cause valence-to-conduction band excitation. These electrons in the illuminated regions will diffuse through the conduction band to the darker regions due to relatively low potential in the darker regions. Leaving behind a net positive charge in the illuminated regions, the migrating electrons will create a space charge distribution, inducing an electric field, \( \vec{E}_{sc} \).

(See Figure 1.2) The magnitude of the induced electric field will vary within the crystal depending on the optical intensity distribution. At steady state there will no longer be any redistribution of charge and the induced \( \vec{E}_{sc} \) will create a change of index of refraction via the electrooptic effect.
Figure 1.2
When a non-uniform intensity pattern enters the crystal the excited electrons migrate and get trapped by the acceptors located in the dark region. The result is a space charge distribution that modifies the index of refraction via the electrooptic effect. [4]
1.3 **Solitons in Photorefractive Material**

While solitons have been shown to form in many non-linear systems from waterways to liquid polymers, the kind of solitons related to the study in this thesis is the so-called photorefractive solitons in nonlinear optics [5]. Although the experiments included in this work do not focus on soliton formation, they do allow for a good model in understanding the properties of photorefractive material, as well as modulation instability.

When creating a soliton in photorefractive material the incoming beam must be gaussian and focused on the input of the crystal. This will cause the dopants located in the high intensity region to be excited into the conduction band, allowing them to move freely through the crystal. Recognizing that there are areas of low potential within the dark regions of the crystal (areas not exposed to light), the electrons will re-distribute themselves into the dark regions. Once the electrons have vacated the illuminated region a net positive charge will be left, while a net negative charge will be formed within the dark region. This will create a space charge distribution that induces a space-charge field between the light region and the dark region. These properties were mentioned previously and are not unique to soliton formation.
One other criterion for soliton formation is the presence of a uniform external static electric field across the crystal. This externally applied field will be partially screened by the non-uniform induced space-charge field created from photo-excitations. Consequently, the magnitude of the total electric field will be lower in the regions of higher optical intensity and higher in the regions of low optical intensity. The result is a crystal in which the index of refraction is lowered in the darker regions through the electrooptic effect, while there is little change of index in the illuminated regions because of the screening process. The change of index will mimic the intensity profile of the beam, creating a lensing effect in which light can become focused. If all conditions are met this lensing effect will balance the inherent diffraction of light, thus inducing a stable wave-guide, now commonly known as a “soliton”. These solitons have been demonstrated experimentally about half dozen years ago, including both bright [6] and dark [7] solitons. Introduction to other types of optical solitons such as Kerr solitons (using optical Kerr effect) and quadratic solitons (using X2 effect) can be found in the review article of G. Stegeman and M. Segev in Science, volume 286. [5]
Figure 1.3
Formation of a photorefractive screening soliton
A) A non-uniform beam enters the photorefractive material which creates an electric field. B) The uniform background beam partially screens the electric field. C) The index of refraction is diminished in the illuminated region of the crystal. D) The light tends to stay where there is low resistance and therefore experiences trapping.[8]
1.4 **Modulation Instability**

Most non-linear wave systems exhibit an instability that will lead to small changes in the wave’s amplitude or phase due to perturbations encountered within the system. This phenomenon is called modulation instability (MI) and is caused by the simultaneous occurrence of dispersion and non-linearity in these systems. In an optical system MI causes a beam with uniform intensity to disintegrate and break up into filaments (or pockets of light). As the wave propagates through the non-linear medium the intensity of these filaments grows at an exponential rate.

MI was often the cause for much frustration in experiments involving soliton formation, making the wave-guide unstable and disintegrating the soliton into filaments. Presently, it is believed that MI is actually a precursor to soliton formation. In an article recently published in Science Magazine the researchers stated, “The relation between MI and solitons is best manifested in the fact that the filaments (or pulse trains) that emerge from the MI process are actually trains of almost ideal solitons.” [5]
1.4.a Why focus on incoherent MI?

In the early years of experimental non-linear optics it was the high intensity lasers that opened the door to proceed with this research. Consequently, the coherent light produced by lasers has been the main tool used to study non-linear properties, including solitons and MI, and incoherent light was thought to have insufficient properties to support these non-linear phenomena.

Not until 1996, the first demonstration of optical solitons using incoherent light was reported [9]. Shortly thereafter, the demonstration of white light solitons was reported in Nature [10], and the demonstration of incoherent dark solitons was reported in Science [11]. Since then, incoherent solitons have been studied extensively, both experimentally and theoretically. It seems quite natural to investigate their close relative, incoherent MI. What has been theoretically predicted [12], and shown experimentally in this thesis [13,14], is that incoherent MI has one characteristic that does not at all exist in coherent MI. Incoherent MI only occurs when the non-linearity exceeds a threshold that is defined by the spatial (or temporal) coherence of the wave. Therefore, MI can be suppressed if the wave has a low degree of spatial coherence, whereas coherent waves have no threshold for MI to occur. This unique property of incoherent MI has led to observations of a number of fascinating
nonlinear phenomena mediated by incoherent MI, including incoherent anti-dark solitons [15], incoherent pattern formation [16, 13], soliton clustering [17], and incoherent photonic latticed [18].
1.5 Coherent vs. Incoherent Light

All sources of light have some degree of spatial coherence. The light emitted from a white light source (i.e. a light bulb) is at the highly incoherent extreme of spatial coherence, while a laser is at the other extreme, being mostly coherent. One can think of a white light source emitting a light beam with random phase fluctuations across the whole beam in space (spatial incoherence) and with many varying wavelengths all at random intervals (temporal incoherence). Laser light is produced when a group of atoms are pumped into an excited state and upon their de-excitation light is emitted collectively, thus making the output coherent, all in phase and of the same wavelength. [19]

1.5.a Characterizing the level of coherence of a light source

The spatial coherence of a light source can be measured by the rate of diffraction exhibited by a beam as it propagates. This rate of diffraction is related to as the coherence length, $\ell_c$. A white light source can have a coherence length close to a micron, indicating that as the beam travels one micron it will spread considerably. However, a laser beam, having a long coherence length, can travel over several meters without much spreading of the beam. [20]
The idea of spatial coherence can be better understood by going back to an early classical experiment by Thomas Young. Consider a coherent light source that passes through a mask with one slit and then proceeds to a viewing screen. A fringe with high contrast would be observed on the screen. If a second coherent source, identical to the first but out of phase with it, were introduced through a second slit, a second fringe pattern would be generated. This second fringe would be the same only shifted with respect to the corresponding position of the first fringe. If the two slits were close together the resulting fringe pattern would be rough but still distinguishable. (See Figure 1.4.a) If the slits were placed farther away from each other, the two fringes would begin to partially cancel one another resulting in a single smeared fringe. (See Figure 1.4.b) This is because the spatial correlation between each source becomes weaker as more distance is placed between the slits. If the mask had many slits with many sources that are coherent but out of phase, there would be a total loss of visibility of the fringe. We consider the light produced by these many slits as spatially incoherent light. [22]
Figure 1.4
a) Two coherent sources pass through two slits that are close together and the resulting interference pattern is still prevalent. b) As the slits move farther apart, the same coherent sources will have interference patterns that start to cancel. The lack of a fringe pattern indicates that the sources simulate a partially incoherent beam. [22]
Chapter 2

2.1 Creating Partially Incoherent Light Using a Diffuser

One way to achieve a spatially incoherent light source is to pass the coherent beam through a diffuser. The diffuser causes the light to form a multitude of bright and dark spots that create an unordered pattern referred to as “speckles”. A typical speckle patterns from our diffuser can be seen in Figure 2.1. Because of the speckles, the beam will experience much scattering. Even though there is an extremely complex change in the amplitude and phase of the wave, the distribution of these speckles is constant in time and therefore the coherence of the beam is not reduced. In order to make the beam substantially incoherent, the profile of the speckles must change randomly with time. For that reason a rotating diffuser is required. [21]

The rotating diffuser creates a time averaged intensity pattern that is much faster than what human eyes can detect, essentially creating a uniform beam. In view of this fact it is important to note that the photorefractive SBN-60 crystal behaves as our eyes do and is non-instantaneous in nature. In other words, nonlinear material that does respond instantaneously will “see” each speckle and form many
small “positive lenses”, causing a fraction of the beam to be captured. Therefore the instantaneous behavior will cause the beam to modulate due to the coherent speckle and give no insight to the perplexity of modulation of a uniform incoherent broad beam.

It is also important that MI can be studied at varying levels of coherence. The configuration developed for these experiments allows us to change the coherence length by simply moving the focal point of the beam to different positions relative to the diffuser. When the diffuser is at a stand still the camera views the speckle pattern formed on the beam. If the diffuser is located close to the focal point of the beam the speckles will appear very large. The large speckles indicate that only a few coherent sources enter the crystal, thereby keeping the beam fairly coherent. As the focal point moves away from the diffuser the speckles will become increasingly smaller in size (Figure 4.1). The crystal therefore experiences many, many coherent sources that are at random phase with one another, resulting in a beam that becomes increasingly spatially incoherent. Once the diffuser is rotating the beam’s time averaged intensity is uniform, but the size of speckle dictates the coherence length.
Figure 2.1
Varying the coherence using a two lens system.
2.2 Experimental Setup

First the argon laser beam is split into two beams having orthogonal polarization, one being the “extraordinarily” polarized beam, or e-beam, and the other the “ordinarily” polarized beam, or o-beam. These names refer to the fast and slow axis or the birefringence of the crystal. Due to the orientation of our crystal on the optical table, the fast axis, which is along the c-axis, is horizontal. Therefore, in our experiments, we consider a beam that is horizontally polarized to be the e-beam and vertically polarized to be the o-beam. [1]

Next, the e-beam is collimated and then focused on a diffuser, causing the beam to quickly diffract. After the beam is passed through the diffuser a lens collects the now incoherent light so that it enters the face of the crystal and propagates along the crystalline a-axis. The o-beam is also collimated before entering the face of the crystal and propagates along the a-axis. The o-beam will be used as background irradiation, which serves two purposes. One is to control the saturation level of the optical nonlinearity. The dopants in our crystal are finite so this allows us to control the number of excited dopants, thereby controlling the effective nonlinearity. Secondly, the background helps to speed up the process of MI formation. This is
because the relaxation time of the crystal depends on the intensity of
the light propagating through the crystal. [4] When monitoring the MI
due to the incoherent light a polarizer is placed in front of the CCD
camera so as to block the background o-beam.
2.2 **Induced Modulation Instability**

In designing the setup to investigate incoherent MI, it is important to isolate properties that contribute to and modify MI gain. For MI to exist there needs to be some perturbation that causes the beam to break up into filaments. The photorefractive crystal used in these experiments contains imperfections that give rise to MI, but these imperfections are not uniform throughout the volume of the crystal. This makes reproduction of experimental results difficult by limiting our control over the noise intrinsic to the crystal. To eliminate this problem we introduced noise to the experiment in the form of a grid pattern. This was beneficial for two reasons. First, we could control many properties of the noise including intensity and spatial frequency. Second, the introduced noise produces MI at lower voltages, eliminating the effects of defects and striations in the crystal.

2.2.a **Cross Phase Modulation**

Two methods are used to add the noise into the system. The first method used is a three-beam interference pattern. The coherent light from an argon laser is split into ordinarily and extraordinarily polarized beams. The ordinarily polarized beam is further split into three beams and recombined at the input of the crystal (Figure 2.2). The motivation is to create two Mach-Zehnder interferometers, which
are amplitude-splitting devices. All three beams travel along separate paths with very small differences in their optical path lengths. It is important the difference is small enough so that each beam remains in phase with its counterpart and allows for well-defined interference patterns. Figure 2.2 shows that beams one and two combine to create a vertical interference pattern and one and three combine to create a horizontal interference pattern. Ideally, beams two and three are significantly out of phase so that no interference pattern forms between them. Experimentally, this proves to be very difficult. These two beams unavoidably create a third component to the grid pattern in the form of a diagonal fringe.
Figure 2.2: One ordinarily polarized beam is split into three separate beams. These beams make two Mach-Zehnder interferometers that create a grid pattern at the input of the crystal.

The ordinarily polarized grid pattern should be clearly seen at the input of the crystal, recombining with the now incoherent extraordinarily polarized beam. The grid pattern will only participate as noise when it passes through the crystal. This is because the electrooptic tensor coefficients of the o-beam and e-beam are $r_{o-beam} = 37 \text{ pm/V}$ and $r_{e-beam} = 280 \text{ pm/V}$ respectively, indicating that the nonlinear effects will be dominated by the e-beam. In addition the grid pattern is set at a low intensity when it is coupled with the
incoherent beam, such that the grid alone, even at high voltages, will not experience any self-focusing as compared to the e-beam. Yet, the small index changes created by the o-beam will affect the e-beam through the process called cross-modulation.

The results of these experiments with the three-beam interference grid pattern showed three predicted results: 1) Induced MI could indeed occur through cross-phase modulation. 2) There is a threshold below which incoherent MI does not occur. 3) Grid spacing has a significant effect on the possibility that MI gain occurs. These preliminary results are interesting though there are questions about the consistency of the grid pattern and how variations of the grid pattern can affect incoherent MI. Some of these questions include: 1) is the intensity and width of the vertical fringe equal to the horizontal fringe? In using attenuators to modify the beam intensities there was some control of fringe intensity, but the fringe quality also depended on the relative phase and optical path of the interfering beams. If one orientation had a more defined fringe pattern it would be more likely to create MI in this direction. Similarly, if the width of the fringe in one orientation had a preferential spatial frequency then this too would promote strong MI. 2) Does the diagonal fringe contribute to stabilize MI in one direction and promote MI in another? And could this contribution vary as the diagonal fringe becomes more vertical or
horizontal? 3) In order to obtain clear fringes in each orientation the beams often do not enter the crystal perpendicular to the face. The beams may enter the crystal such that there is a clear grid pattern at the face but due to the angle of entry the induced noise either washes out MI or disproportionately amplifies MI. It is these doubts that caused me look for a more accurate way of introducing noise into the system, so that more consistent data can be compared with already completed data.

2.3.b Photomasks

The second method of introducing noise to the experiment is the use of photoslides. The photoslides are made from argon chrome soda lime masks with varying grid spacings including 25, 40, 50, 65, 75, 85, 100, 110, 150 and 200 microns. Once the desired coherence is obtained with the diffuser configuration, a mask is placed in the path of the incoherent beam at a position where the grid is seen clearly at the input of the crystal. This provides a spatial modulation directly on the incoherent beam with the grid quickly fading into a uniform beam as it moves through the crystal.
As the reader will see, both of these approaches were valuable in convincing us of several theorized results. One finding is that incoherent light does have a threshold below which MI is suppressed, whereas this threshold does not exist in the coherent realm. Additionally, the threshold below which there is no MI gain is directly related to the coherence length of the beam. And finally the result that I found most intriguing is that the spacing of the perturbations contributes substantially to the MI gain, and this too is related to the coherence length.
2.4 Threshold for Incoherent MI

As mentioned in Chapter 1, it will be shown that incoherent MI does occur in nonlinear systems, but only if the strength of the nonlinearity exceeds a threshold that is determined by the spatial correlation or coherence of the beam. The threshold has been calculated theoretically such that the gain coefficient can be represented by:[23]

\[ g(\alpha) = -q_o |\alpha| + |\alpha| \sqrt{k_o} - (\alpha/2)^2. \]

where \( q_o \) is the coherence coefficient, \( k \) is the wave vector, \( \alpha \) is the spatial wave vector and \( I_o \) is the uniform background intensity. This equation indicates that when \( g(\alpha) > 0 \) the beam will experience MI causing a uniform beam to break up into filaments. Otherwise, when \( g(\alpha) < 0 \) the perturbation that causes MI will decay and no patterns will grow on top of the uniform input intensity. This further relates to the coherent limit where as \( q_o \to 0 \) (i.e. a fully coherent beam) the growth coefficient is always positive and MI will always occur. Alternatively, as \( q_o \to \infty \) (i.e. a fully incoherent beam), the growth coefficient will be increasingly negative and no MI gain will occur. This is referred to as incoherent wash out. In addition, there exists a threshold where \( q_o = k_o \) that determines the occurrence of MI; if
$q_o < \kappa d_o$ MI occurs, whereas when $q_o = \kappa d_o$ MI is completely washed out. Simply put, for a fully coherent beam MI will always occur in the presence of a perturbation, but for an incoherent beam there exists a threshold below which MI will not occur if the spatial incoherence washes out the effects of the nonlinear self-focusing. [23]

This concept can also be described intuitively if you consider a uniform coherent beam with a perturbation superimposed onto the beam. When this beam enters a linear medium the perturbation will neither grow nor decay as the beam propagates. There will be no change in how the beam propagates through the crystal because of the linear properties of the material and because the beam varies in unison with time. Once any self-focusing nonlinearity is added to the system, the intensity pattern of the perturbation will cause an index change. This results in self-focusing within the medium such that the uniform beam will begin to break down depending on the intensity pattern of the perturbation. Therefore, one can conclude that coherent MI occurs even if the nonlinearity is increasingly small, because there is no mechanism that can suppress the exponential decay of the uniform beam.[23]

Now consider a partially incoherent beam entering a linear medium. Again there is a perturbation superimposed on the beam. As the beam propagates through the crystal the perturbation will fade
and the beam becomes more uniform. This is because the partially incoherent beam behaves as if there were many separate sources entering the crystal, making the modulation visibly wash out. By adding the same self-focusing nonlinearity as before the system will acquire some self-focusing properties, but whether MI occurs depends on two conflicting effects: the washout effect arising from the level of incoherence, and self-focusing effects of the nonlinear system that amplify the existing perturbation. It is this battle between the two above-mentioned conflicting effects that gives rise to the threshold only above which incoherent MI occurs. [23]
3.1 Cross-phase Modulation with 3-Beam Interference Pattern

As discussed in section 2.3, we began by using a 3-beam interference pattern. The first step is to split the coherent source into extraordinarily and ordinarily polarized beams. The ordinarily polarized beam, or o-beam, is split into three with the use of 50-50 beam-splitters and mirrors to create two Mach-Zehnder interferometers. The three beams are then recombined at the input of the crystal. Attenuators allow us to adjust any discrepancies between the intensities of the beams. The optical path difference of all of these beams must be small enough to achieve well-defined fringe patterns. The three beams are manipulated to make the horizontal and vertical fringe patterns that form a grid pattern, which is clearly visible at the input of the crystal. The experimental setup that was used to create the two Mach-Zehnder interferometers can be seen in Figure 2.2.

The extraordinarily polarized beam, or e-beam, is passed through a rotating diffuser to generate a spatially incoherent beam. The coherence of the beam depends on the size of the speckles when the diffuser is stationary. Before each experiment many speckle sizes in an image are measured and averaged to obtain an approximate
coherence length, $l_c$. At this point the o-beam and e-beam are coupled at the input of the crystal so that the spatial noise is seeded through cross-phase modulation. Each beam has a wavelength of 488nm, and they co-propagate along the crystalline a-axis. The photorefractive crystal (SBN:60) with dimensions of 5mm × 5mm × 20mm is used as the nonlinear medium for these experiments. An applied voltage is connected across the c-axis in order to control the nonlinear properties of the crystal. An imaging lens magnifies a small portion of the crystal that we view using a solid state CCD camera, then the analog data is converted to digital information using software made by the company Coherent, called “Beam View Analyzer”. This set up in its entirety is illustrated in Figure 3.1.
Figure 3.1
Experimental setup for induced MI through cross-phase modulation.
3.2 Experimental Results

3.2.a. Testing Induced MI

In designing the experiment it is important that the ordinary beam that carries the grid pattern does not cause self-focusing alone. If this were the case then we would not be observing induced MI but rather guiding of incoherent beam, as instead we would have created in essence an array of solitons. In order to test the non-linear effects on the grid pattern, high voltages are placed across the crystal and the grid and incoherent beam are allowed to come to steady state. Blocking the incoherent beam using a polarizer, we could observe the effects of the grid during the process. As is shown in figure 3.2, the grid has no self-focusing properties when observed alone. Image 3.2.a shows the speckles when the rotating diffuser is stationary, signifying the coherence length of the uniform incoherent beam. Image 3.2.b and 3.2.c are the result of introducing voltages across the crystal with values of 400 volts and 800 volts respectively. In both cases the beam remained fairly uniform. The second column, starting with 3.2.d, shows the grid spacing and intensity that was used for inducing MI. Images 3.2.e and 3.2.f show the incoherent beam (the grid being blocked using a polarizer) at 400 volts and 800 volts respectively, exhibiting that the grid introduces MI to the system. The final column shows the grid only (changing the polarize so that now the incoherent
beam is blocked), with uniform intensity amplification applied to enhance effects. Image 3.2.g shows the beam at the input of the crystal when there is no voltage across the crystal. Image 3.2.h shows the grid when 400 volts are applied and 3.2.i shows the grid when 800 volts are applied. The grid pattern remains almost entirely unchanged. Thus we concluded that the grid only provides noise to the system to induce MI and does not contribute to the nonlinear effects alone.

Figure 3.2
a) Speckle size, $l_c = 30$ microns, b) and c) incoherent beam only, b) $V=400$, c) $V=800$, d) grid intensity and spacing, e) and f) induced MI through cross-phase modulation, e) $V=400$, f) 800, g), h) and I) grid only, g) $V=0$, h) $V=400$, I) $V=800$. 
3.2.b. Grid Induced vs. Self-Induced Threshold

Secondly, we wanted to show that incoherent MI has a threshold in which the more incoherent the beam the higher the voltage required to observe MI within the crystal. Furthermore, when the seeded noise is introduced incoherent MI occurs at a voltage lower than one that is needed for self induced incoherent MI from the intrinsic noise of the crystal. We began by choosing a coherence in which the beam does not experience MI at a certain voltage, but at higher voltages the beam will begin to breakup.

Typical experimental results can be seen in Figure 3.3. The images 3.3.a and 3.3.b show the speckle size when the diffuser is stationary and the grid pattern that is used to introduce noise. The coherence length of this speckle size is approximately $l_c = 25$ microns. Image 3.3.c is the incoherent beam alone when 400 volts are passed across the crystal. At steady state the beam has almost no break-up. This is the threshold; above this level of non-linearity the beam begins to experience MI, but at 400 volts the beam remains fairly uniform. As the voltage is increased to 500V, just past the threshold, striations and defects in the crystal create the noise needed to cause noticeable breakup, as can be seen in 3.3.d.

At this point the grid pattern, shown in Figure 3.3.b, is introduced to the system. Again the voltage is increased to 400V and
the crystal is allowed to come to steady state. The beam starts to breakup at this lower voltage of 400, with the grid dominating the pattern formation, as can be seen in Figure 3.3.e. It is clear that the grid was the direct cause of the incoherent MI and when the incoherent beam is blocked the grid shows no signs to self-focusing. Finally, if the grid is once again removed from the system and the crystal achieves steady state, the incoherent beam will return to the previously observed uniform state.

Figure 3.3
a) Coherence length, \( l_c = XX \) b) grid pattern (intensity enhanced), c) incoherent beam at V=400 (no grid), d) induced MI at V=400 (with grid blocked), and e) MI of incoherent beam at V=500 (no grid).
3.2.c) Increased Non-linearity

The next experiments build on the induced incoherent MI, testing the dependence on the strength of the non-linearity. The DC field placed across the c-axis, as described previously, controls the strength of the non-linearity. For these experiments, the intensity ratio of the beams is set at a constant and the coherence length remains unchanged. With each increase of the voltage, the crystal is allowed to come to a steady state and an image is taken at the output of the crystal.

Typical experimental results are shown in Figure 3.4. It can be seen that at a low non-linearity the beam remains uniform; as the non-linearity is increased, the beam breaks up into 1-D, 2-D and disordered patterns. In this figure, image 3.4.a and 3.4.b show the speckle size associated with the incoherent beam, \( l_c = 20 \) microns and the grid pattern is introduced to the system for noise. Images 3.4.c through 3.4.g show the incoherent beam at the output of the crystal when the voltage is set at increasing intervals from \( V_c = 200 \) volts up to \( V_g = 1400 \) volts. When the non-linearity is under the threshold \( (V_c = 200V) \) the incoherent beam shows no sign of MI gain. At a voltage close to the threshold \( (V_d = 500V) \) induced incoherent MI occurs and leads to breakup of the beam into 1-D stripes. As the voltage is further increased \( (V_f = 1100 \) volts), MI causes the beam to break up in
two dimensions and to form structures akin to arrays of solitons. Well above the threshold \( V_g = 1400 \text{V} \) the beam has a distorted pattern formation.
Figure 3.4
Keeping a constant coherence length and intensity ratio the non-linearity is increased from $V=200$ to $V=1400$ in regular intervals of $300V$. As you can see the induced MI goes from uniform, to 1-D break-up, to 2-D break-up, and finally a disordered pattern forms.
3.2.d. Increasing Coherence Length

The final series of experiments completed with the three-beam interference pattern shows the relationship between induced MI and the level of coherence. The goal is to test the theory that there is a threshold above which incoherent MI will not exist, and that this threshold is a function of the coherence of the beam (8). Using various coherence lengths (the smallest speckles producing the most incoherent beam, while as the speckles become larger the beam becomes more coherent), we can test the existence of a threshold to incoherent MI. For these experiments, the range of coherence was from approximately $l_c = 5$ microns, a spatial incoherence on the same order of magnitude as white light, to $l_c = 60$ microns, nearing a coherence at which there is no threshold for MI.

Typical experimental results are shown in figure 3.5. The first column in the figure shows the speckle size when the diffuser is stationary, increasing in size and therefore coherence from Row A through Row D. The second column shows the two dimensional fast Fourier transform, 2-D fft, of the corresponding speckle image. As the speckles become larger, the widths of corresponding fft spectrum become smaller. In general, the random speckle pattern does not have a well-defined frequency in Fourier space. But as the beam
becomes more coherent, with increasing in the size of the speckles as well as the spacing between speckles, the average spatial frequency, or the width of the power spectrum will become smaller. Testing the most incoherent source first, we find a voltage at which the beam remains uniform, as can be seen in the third column of Row A. Keeping the voltage constant and increasing the coherence, the beam starts to break up in a 1-D pattern induced by the grid. When the coherence is increased the washout effect of the incoherent light begins to diminish and the beam breaks down further into 2-D filaments. Finally, when the beam becomes substantially coherent the threshold is well below the maintained voltage and a disordered pattern forms.

These results further reinforce the theory of incoherent MI: when the beam is sufficiently incoherent, MI does not occur; the non-linearity is below the threshold. As the beam becomes more coherent the non-linearity approaches the threshold and 1-D, 2-D, and disordered patterns will form. The MI that forms when the beam is quite coherent has filaments that are smaller in size than the speckles themselves. This indicates that when the threshold approaches zero, $V \rightarrow 0$, the highly coherent beam breaks up as determined by the intrinsic noise such as from the crystal’s striation, and we no longer are looking at induced incoherent MI. Since the coherent MI has co
threshold, where the beam is made too coherent while the
nonlinearity is kept at the same high level, the breakup tends to be
very strong.

Figure 3.5
The first two columns show the coherence length and its
corresponding 2-D FFT. The last column shows the induced MI when
the non-linearity is set to V=500 volts. Row A: $l_c=10$ m, Row B:
$l_c=20$ m, Row C: $l_c=30$ m, and Row D: $l_c=60$ m.
3.3 Conclusion

To try to quantify this trend, many experiments were completed in which the threshold voltage was found for varying coherence lengths. When analyzing the data, we found that there were two intrinsic sources of error that arose. The first was calculating the coherence length by measuring the speckles. The speckle sizes are not at all uniform in any region of the diffuser, although when the speckles are small there is little variation in size, allowing a more accurate calculation of coherence. As the speckles become larger there is a much greater deviation in the sizes and therefore a greater error in determining the coherence. Keeping this in mind, when the beam is quite spatially coherent (i.e. large speckles) it breaks up at a low, but very distinct, voltage (uncertainties of no more than 5 volts). As the beam becomes more incoherent the exact threshold voltage becomes vague (uncertainties up to and greater that 50 volts). It is the observer that must determine the threshold coherence as well as the threshold voltage values and the amount of uncertainty that can be attached to each value. For this reason, with this experimental data, only a general trend in which the threshold voltage depends on the coherence length can be realized, shown in Figure 3.6
FIGURE 3.6
Measured threshold values of externally applied voltage as a function of coherence length.
Chapter 4

4.1 Seeded Noise Using Various Photomasks

One of the more interesting results that arose from the three-beam interference grid was evidence showing the dependence of induced incoherent MI on the spacing of the grid pattern. Because the three-beam interference was difficult to reproduce, our attempt at proving the dependence of MI on grid spacing was never realized. In this chapter we explain a new technique in which we could vary the spatial frequency of the perturbations while still having control of the nonlinearity of the crystal as well as the intensity and coherence of the beam. Alongside previous observations in which induced incoherent MI depends on the coherence of the beam and the strength of the nonlinearity, we were also able to demonstrate that induced incoherent MI depends strongly on the perturbation period (or spatial frequency).

In past experiments incoherent MI was studied using two different approaches. The first was to allow the noise to be driven by intrinsic properties of the crystal such as defects and striations in the crystal, completed in [16]. The second method was seeding the noise through cross-phase modulation, completed in our previous
experiments [13], as discussed in Chapter 3. For the following set of experiments an alternative approach was used, one in which seeded noise was introduced directly to the incoherent beam with the use of amplitude masks. This allowed us to spatially modulate the beam with different grid spacing, therefore changing the perturbation period produced by the noise. By actively seeding spatial noise onto an otherwise uniform incoherent beam, we demonstrate that the induced modulation instability has a maximum growth at a preferential perturbation period, leading to formation of ordered patterns.

In order to study this phenomenon photomasks were used, all of which have the same transmission intensity but varying spacing of the grid patterns. Ten masks were made in all, with the spacing varying from 25 microns to 200 microns, some of which can be seen in Figure 4.1. Similarly to the experiments discussed in Chapter 3, the argon ion laser beam (\(\lambda=488\text{nm}\)) is split into o-beam and e-beam. The e-beam is passed through a rotating diffuser, to produce a partially spatially incoherent beam. Changing the focus of the beam on the diffuser controls the spatial coherence; the more focused the beam the larger the speckle, resulting in a more coherent beam, and vice versa. When the rotating diffuser is stationary the speckle size is averaged and this value corresponds to the coherence length, \(l_c\).

Once the desired coherence is obtained and the diffuser begins
rotating, the mask is placed in the incoherent beam’s path such that a well defined grid pattern is seen at the input of the SBN:60 crystal. The incoherent beam will be extraordinarily polarized and have a spatial modulation corresponding to the mask used, thus providing the seeded noise. The incoherent beam will dramatically diffract as it propagates through the crystal causing the grid pattern to quickly fade into an almost uniform beam at the output. In addition to the incoherent beam, a uniform ordinarily polarized coherent beam will illuminate the crystal, allowing us to fine-tune the nonlinearity of the crystal through dark illumination (as discussed in Chapter 2). Again we use a bias DC field to alter the nonlinearity of the crystal so we can enable ourselves to work in a regime close to the induced MI threshold. This means that without the seeded noise the incoherent beam would remain uniform, being at a voltage below the threshold in which MI forms, but with the grid pattern imposed on the beam we are able to induce MI. The setup is illustrated in Figure 4.2.

![Figure 4.1](image)

**Figure 4.1**
Grid spacing produced by photomasks, measured in microns. a) 40, b) 50, c) 65, d) 75, e) 85, f) 100, g) 110.
Figure 4.2 Experimental Setup with photomasks
4.2 Experimental Results

4.2.a) Testing Induced MI

This method of introducing spatial modulation directly to the incoherent beam had not been investigated before, so first we wanted to see if it exhibited properties similar to incoherent MI induced by cross-phase modulation. Similarly to our previous experiments, we first kept the grid spacing, beam coherence, and intensity ratio fixed, and then increased the nonlinearity by increasing the bias field. As Figure 4.3 shows, using the photomasks produces almost identical results as with cross-phase modulation. The spatial coherence of the beam from this set of data is fixed at 25 microns, the grid spacing is fixed at 110 microns, and the intensity ratio is approximately 3 to 2. The beam, in this case, stays uniform below the threshold voltage of 500V and then begins to break up into 1-D stripes at 750 volts. Increasing the voltage even more leads to 2-D breakup above 1000 volts, showing a consistency with the cross-phase modulation data.
Figure 4.3
Fixed coherence ($l_c = 25$ microns), fixed spacing (110 microns) increasing voltage (measured in volts), a) 500, b) 750, c) 1000, d) 1200, e) 1500, f) 2000.
4.2.b). Dependence on Coherence

The second test was to keep the spacing, nonlinearity and intensity ratios constant and vary the coherence of the beam. Figure 4.4 shows the progression from uniform to 1-D, and then to 2-D patterns. The first column shows the speckle sizes, starting with the most incoherent and increasing until the beam is almost totally coherent. The second column shows the output intensity at a voltage of 760V with a perturbation period of 70 microns. The third column shows the 2-D fft of the induced MI. When the beam is very incoherent the fft image shows no breakup. As the coherence increases the fft illustrates the 1-D to 2-D progression. These observations are also in agreement with observations and theoretical predictions made in the 1-D case [24]. Furthermore, they are consistent with previous experiments of incoherent MI dominated either by preferential noise along the direction of crystal striations or by noise seeded through cross-phase modulation.
Figure 4.4
Fixed grid spacing and fixed nonlinearity (V= 760 volts), increasing coherence. Column one shows speckle size, column two is the incoherent beam at crystal output and Column three is 2-D fft of incoherent beam to show beam breakup from uniform to 2-D. Coherence length measured in microns: a) $l_c = 5$, b) $l_c = 10$, c) $l_c = 20$, d) $l_c = 30$, e) $l_c = 50$. 
4.2.c) Dependence on Spatial Frequency

The final experiments involved changing the spatial frequency of the noise and looking for a correlation with incoherent MI. For this data set the coherence of the beam was kept constant at a coherence length of approximately 15 microns, as averaged from the speckle size when the diffuser is stationary, and the voltage across the crystal was maintained at 750V. While observing the input of the crystal, the photomasks were placed in the line of the incoherent beam so a clear grid pattern could be seen having a peak intensity that remains constant for all grids. The ordinarily polarized background beam was also kept at a constant intensity such that the intensity ratio between both beams is also constant. The first column in Figure 4.5 shows the grid pattern at the input of the crystal indicating the spatial modulation. The second column is the image in real space at the output when the voltage is on and the crystal has come to a steady state. The third column is the 2-D fft of the output intensity data. The line graphs above each image in the second and third columns are intensity profiles of the image and the 2-D fft to visually compare the peak intensity versus the spatial frequency of the perturbation. While it is difficult to quantify the MI gain, one can readily see, in comparison, which spatial frequency provides the most gain.
From top to bottom the perturbation period is increased, as shown in column one. At a spacing of 40 microns the beam remains fairly uniform. This is because the spatial period approaches the speckle size, thus further decreasing the coherence length experienced by the crystal, which washes out the MI gain. As the spacing is increased to 65 microns the intensity pattern of the beam starts to break up into 1-D stripes. This 1-D breakup becomes even more prevalent when the periodic spacing is increased to 85 microns. Looking at the corresponding spatial power spectra the amplitude can be seen to increase in Fourier space, indicating that these spatial frequencies induce a greater MI gain. The 100 micron grid produces an even greater induced MI gain, causing the beam to break up into 2-D filaments. This can be seen from the power spectrum where the spatial frequency has both horizontal and vertical components. In fact, the amplitude peaks at 100 microns, indicating that this spatial frequency has the maximum MI gain for this coherence length.

Increasing the perturbation period (or decreasing the frequency), the induced modulation instability of the beam begins to fade into a 1-D pattern at 110 microns, and then a fairly uniform pattern reappears when the grid spacing is increased further to 150 microns. This shows that the perturbation period that induces incoherent MI has a
gain curve that is dependent on spatial frequency, as predicted in theory by Kip et al. [25]
Figure 4.5
Induced MI of a partially incoherent beam with varying spatial perturbation periods. Shown are photographs of intensity patterns taken at crystal output (left), along with their corresponding spatial power spectrum (right). All data were taken under the same experimental condition (coherence: 15µm; fixed voltage: 750 volts), except that the period of perturbation was varied. From top to bottom, the periods of perturbation are 40 µm, 65 µm, 85 µm, 100 µm, 110 µm and 150 µm.
Similar results can be seen in Figure 4.6. Although for this figure the grid spacing is included to further show the induced MI. The coherence length of the beam is increased so that MI occurs at a lower voltage than the previous set of data. This is not the only effect of the increased coherence; the beam also breaks-up when the perturbation has a smaller spacing (or higher frequency).

The first columns show the perturbation period of the grid pattern, in increasing order from 40 microns to 110 microns. Column two shows the image of the partially incoherent beam at the output of the crystal. The nonlinearity is fixed at 600 volts and the coherence length is doubled to approximately 30 microns. Finally, the third column shows the level of induced MI through 2-D fft.

As the reader can see, the beam experiences its peak MI at a much smaller perturbation period of 50 microns. When the perturbation period is increased (spatial frequency is decreased) the MI is slightly washed away.
Figure 4.6
Fixed nonlinearity (V= 600 volts) and fixed coherence length (\(l_c = 30\) microns). Column one shows grid spacing, Column two beam at output and Column three is the 2-D fft of the incoherent beam. Grid spacing measured in microns: a) 40, b) 50, c) 60, d) 75, e) 85, f) 100, g) 110.
4.3 Conclusion

In summary, we have demonstrated nonlinear propagation and modulation instability of a partially spatially incoherent beam driven by seeding perturbation and the noninstantaneous self-focusing nonlinearity in the photorefractive medium. Main features of incoherent MI and novel pattern formation via symmetry breaking are observed in our experiments. Since nonlinear systems involving partial coherence, weak correlation or symmetry breaking are abundant in nature, our results may prove relevant to other fields of nonlinear physics.
Reference


