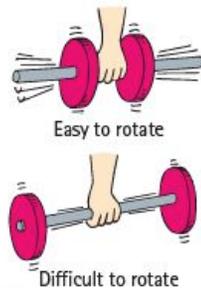


Rotational Inertia (Moment of Inertia)

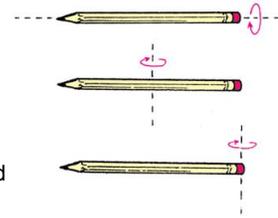
- Rotational inertia depends on
 - Total mass of the object
 - Distribution of the mass relative to axis
- Farther the mass is from the axis of rotation, the larger the rotational inertia.
- Rotational inertia \sim (mass) \times (axis_distance)



Rotational Inertia (or Moment of Inertia)

Depends upon the axis around which it rotates

- Easier to rotate pencil around an axis passing through it.
- Harder to rotate it around vertical axis passing through center.
- Hardest to rotate it around vertical axis passing through the end.



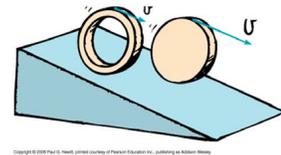
Calculating Moment of Inertia

- Point-objects (small size compared to radius of motion): $I = \sum m_i r_i^2$
- Solid sphere (through center): $I = 2/5 MR^2$
- Hollow sphere (through center): $I = 2/3 MR^2$
- Solid disk (through center): $I = 1/2 MR^2$
- Hoop (through center) : $I = MR^2$

See textbook for more examples (pg. 314)

Example: Hoop vs. Disk

- Imagine rolling a hoop and a disk of equal mass down a ramp. Which one would win?
- Which one is "easier" to rotate (i.e., has less rotational inertia)?



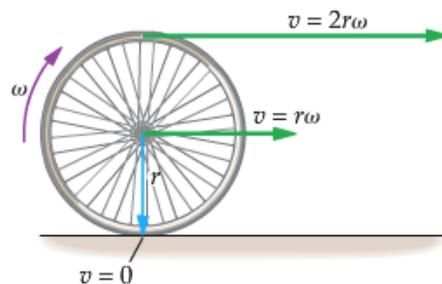
Rolling without slipping

- If the object completes one rotation, its center will move a linear distance of exactly one circumference:

$$\Delta x = 2\pi r$$
- This gives us a relationship between linear velocity (of the center of the object) and angular velocity:

$$v = 2\pi r / \Delta t = r\omega$$

Rolling without Slipping:

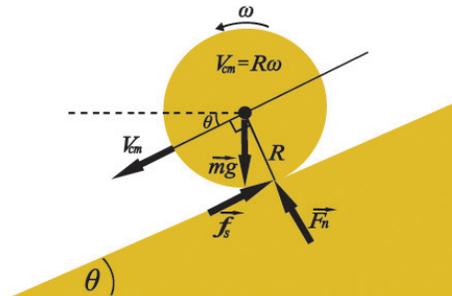


Example: Spinning your tires

Consider a spot on the top of a car tire with radius 65 cm. If the car is stuck in the mud and the wheels just spin, what is happening? What is different when the tires get traction and start to roll without slipping?

- Location of pivot point
- Speed of the point at the top of the tire

Rotational KE: $KE_r = \frac{1}{2}I\omega^2$



A solid, spherical rock rests at the top of a 3.0 m hill and rolls (without slipping) down to the bottom. What is the final speed of the rock at the bottom of the hill?

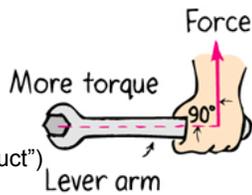
$I = \frac{2}{5} mr^2$ for a solid sphere rotating through center

Rolling without slipping

- Applications: string unwinding from a cylinder or pulley
- Wheels on the road: connect linear motion and rotation
- Careful: don't apply $v = r\omega$ blindly, or for any old location on the rolling object! Ask yourself if it makes sense for the problem.

Torque

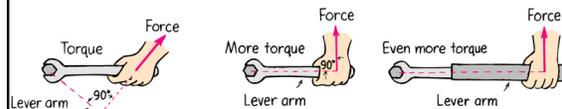
- Torque is the rotational analog of force.
- Same sign convention as other rotational quantities
- Depends on:
 - Magnitude of Force (F)
 - Direction of force
 - Lever arm (r)



- $\tau = r \times F$ (← "cross product")
- Units: Nm

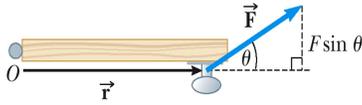
Examples of Lever Arm

- Lever arm is amount of perpendicular distance to where the force acts: points from pivot point to location of force



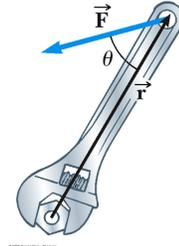
Torque

- Forces are not always perpendicular to the lever arm! Torque definition “picks out” the perpendicular **component** of the force.
- $\tau = r_{\text{perp}} F$ or $= r F_{\text{perp}}$
- Torque is a vector quantity, can be treated in the same way as forces.



Right Hand Rule for Torque

- Point the fingers in the direction of the position vector
- Curl the fingers toward the force vector
- The thumb points in the direction of the torque



Torque Example

A plumber attempts to loosen a bolt by pushing straight downward on a wrench. If she can exert 75 N of force, compare the torques in two scenarios: one where the wrench is at an angle of 30° above the horizontal, and the other where the wrench is completely horizontal.

Which torque is stronger?

How much stronger is it (by what factor)?

Revisiting Newton's Laws

1: Need a linear force to change an object's linear motion \rightarrow Need a torque to change an object's rotational motion

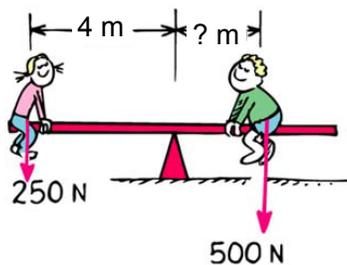
• Equilibrium:

– Linear: $\Sigma F = 0$

– Rotational: $\Sigma \tau = 0$

2: Translational acceleration \sim force, and $\sim 1/\text{mass}$ \rightarrow Angular acceleration \sim torque, and $\sim 1/\text{rotational inertia}$

Example: See-Saw Balancing



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