Rotational Motion

In physics we distinguish two types of motion for objects:

- Translational Motion (change of location): Whole object moves through space.
- Rotational Motion - object turns around an axis (axle); axis does not move. (Wheels)

We can also describe objects that do both at the same time!

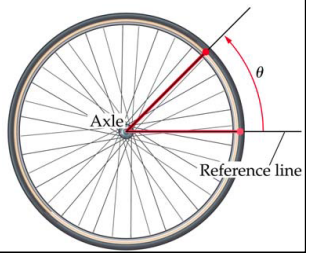
Angular Position

Definition of Angular Position, θ

θ = angle measured from reference line

SI unit: radian (rad), which is dimensionless

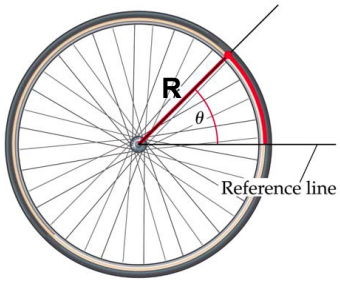
- We will measure angular position in radians:
- Counterclockwise (CCW): positive rotation
- Clockwise (CW): negative rotation



Linear Distance d vs. Angular distance Δθ

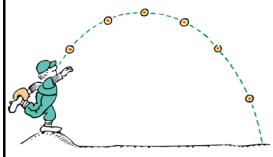
For a point at radius R on the wheel, $d = R\Delta\theta$

ONLY works if $\Delta\theta$ is in radians!

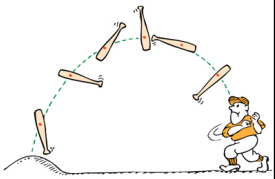


Simple vs. Complex Objects

Model motion with just Position

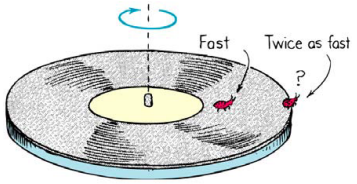


Model motion with position and Rotation



Speed in Rotational Motion

- Rotational Speed ω: rad per second
- Tangential speed v_t: distance per second
- Two objects can have the same rotational speed, but different tangential speeds!



Analogues Between Linear and Rotational Motion (see pg. 303)

Linear Motion with a Constant (Variables: x and v)	Rotational Motion about a Fixed Axis with α Constant (Variables: θ and ω)	
$v = v_i + at$	$\omega = \omega_i + \alpha t$	[7.7]
$\Delta x = v_i t + \frac{1}{2} a t^2$	$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$	[7.8]
$v^2 = v_i^2 + 2a\Delta x$	$\omega^2 = \omega_i^2 + 2\alpha\Delta\theta$	[7.9]

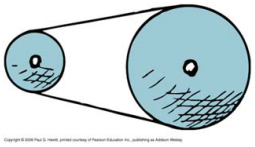
Relationship Between Angular and Linear Quantities

- Displacements
  $s = \theta r$
- Speeds
  $v_t = \omega r$
- Accelerations
  $a_t = \alpha r$

- Every point on the rotating object has the same angular motion
- Every point on the rotating object does *not* have the same linear motion

Example: Gears

- Two wheels are connected by a chain that doesn't slip.
- Which wheel (if either) has the higher rotational speed?
- Which wheel (if either) has the higher tangential speed for a point on its rim?



Angular Acceleration Directions

- If the angular acceleration and the angular velocity are in the same direction, the angular speed will increase with time.
- If the angular acceleration and the angular velocity are in opposite directions, the angular speed will decrease with time.

Corgi on a Carousel

- What is the carousel's angular speed?
- What is the corgi's angular speed? (Relative to ground or carousel)?
- If the corgi's owner spins up the carousel in a time span of 0.5 s, what is the angular acceleration of the carousel?
- Estimate the corgi's tangential speed (hint: how big do you think the carousel is?)

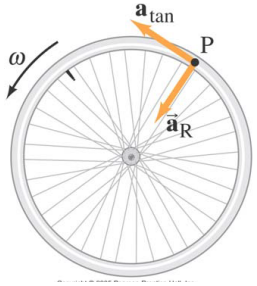
Example: CD drives

In a CD player, the CD spins up from rest to about 500 rpm (revolutions per minute) in a span of about a half a second.

A. What is the angular acceleration of the CD while it is spinning up?
B. What is the change in angle ($\Delta\theta$) that the CD travels through during this time?
C. Once the CD is up to 500 rpm, what is the tangential speed of a point on the outer edge of the disc? What about at the inner edge of the disc?

Angular Acceleration

- Change in angular velocity -> angular acceleration!
- However, even if angular velocity is constant, each point also has centripetal acceleration (due to change in direction of v_t)



Circular Motion & Acceleration

- An object moving in uniform circular motion (ω constant) is still accelerating!
- Acceleration = changes in speed OR changes in direction → Recall from CH 6
- $a_c = v_t^2/R = \omega^2 R$, pointed towards center of circle

Centripetal Acceleration and Angular Velocity

- The angular velocity and the linear velocity are related ($v = \omega r$)
- The centripetal acceleration can also be related to the angular velocity

$$a_C = \omega^2 r$$