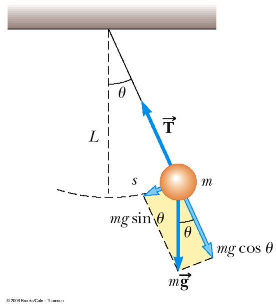
Simple Pendulum

- The simple pendulum is another example of simple harmonic motion
- The force is the component of the weight tangent to the path of motion
  - $F_t = - m g \sin \theta$



Simple Pendulum, cont

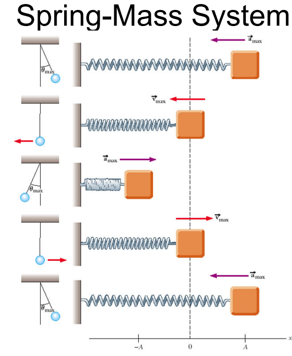
- In general, the motion of a pendulum is not simple harmonic
- However, for small angles, it becomes simple harmonic
  - In general, angles < 15° are small enough
  - $\sin \theta = \theta$
  - $F_t = - m g \theta$
    - This force obeys Hooke's Law

Period of Simple Pendulum

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}} \qquad T = 2\pi\sqrt{\frac{L}{g}}$$

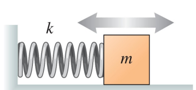
- This shows that the period is independent of the amplitude
- The period depends on the length of the pendulum and the acceleration of gravity at the location of the pendulum

Simple Pendulum Compared to a Spring-Mass System

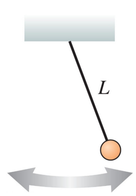


Frequency and Period

The frequency of oscillation depends on physical properties of the oscillator; it does not depend on the amplitude of the oscillation.



$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$



$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

Equations of Motion for SHM:

- Acceleration is NOT constant! We can't use our equations from kinematics.

$x(t) = A \cos (2\pi f t)$

$v(t) = -v_{max} \sin (2\pi f t)$ $\qquad v_{max} = 2\pi f A$

$a(t) = -a_{max} \cos (2\pi f t)$ $\qquad a_{max} = (2\pi f)^2 A$

What if we have friction or drag?

- This is called a "damped" oscillation: the amplitude will steadily decrease over time
- Spring: eventually will resettle back at equilibrium (friction with sliding surface, air resistance, friction at attachments)
- Pendulum: eventually will stop swinging (air resistance, friction at attachments)

Driven Oscillations; Resonance

- Imagine pushing a child on a swingset, applying force on every swing.
  - What if you push at exactly the right time, with exactly the right force one every swing?
  - This "exact right time" is the resonant frequency

Energy Changes

- Mass-on-a-spring: Exchange between kinetic energy ($\frac{1}{2}mv^2$) and potential spring energy ($\frac{1}{2}kx^2$)
- Pendulum: Exchange between kinetic energy ($\frac{1}{2}mv^2$) and gravitational potential energy (mgh)
- This provides a faster way to find the position or velocity of the oscillator at a particular time!

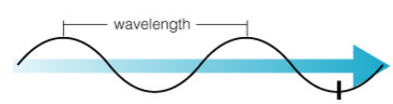
A 0.85-kg mass attached to a vertical spring of force constant 150 N/m oscillates with a maximum speed of 0.35 m/s. Find the following:

A. The period
B. The amplitude
C. The maximum magnitude of the acceleration

Example: A 0.5-kg, 0.75-m pendulum is pulled back at an angle of 10º.

- What are the frequency and period of the pendulum?
- If it just skims the ground at the bottom of the swing, how much energy does the pendulum have?
- How fast is it going at the bottom of its swing?

Properties of Waves

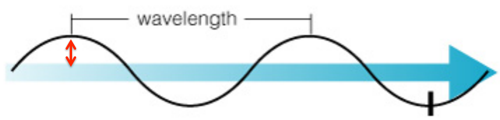


- **Wavelength** is the distance between two wave peaks
- **Frequency** is the number of times per second that a wave vibrates up and down

wave speed = wavelength x frequency

More Wave Properties

- Amplitude: Distance from midpoint to crest of wave



- Period: Time to complete one vibration

Speed of a Wave

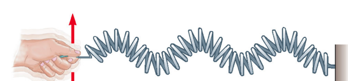
- $v = \lambda f = \lambda/T$
  - Is derived from the basic speed equation of distance/time
- This is a general equation that can be applied to many types of waves
- For sound in air, v = 343 m/s
- For light in a vacuum, $v = c = 3.0 \times 10^8$ m/s

Types of Waves

- Transverse Waves: like ocean waves or waves created by shaking a string
- Longitudinal Waves: compression waves, such as sound

Types of Waves – Transverse

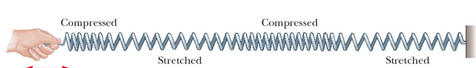
- In a transverse wave, each element that is disturbed moves in a direction perpendicular to the wave motion



(a) Transverse wave

Types of Waves – Longitudinal

- In a longitudinal wave, the elements of the medium undergo displacements parallel to the motion of the wave
- A longitudinal wave is also called a compression wave



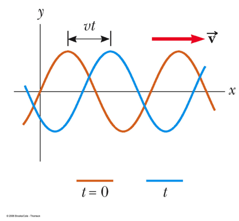
(b) Longitudinal wave

What exactly is doing the waving?

- Many waves require a medium: substance or object to move around
  - Sound waves
  - Water waves
  - Waves in a string
  - Waves in a spring/slinky
- Waves transmit energy without moving the entire medium through space

Waveform – A Picture of a Wave

- The brown curve is a "snapshot" of the wave at some instant in time
- The blue curve is later in time
- The high points are *crests* of the wave
- The low points are *troughs* of the wave



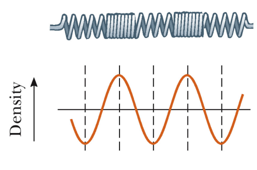
Sinusoidal Waves

- A wave can take on a variety of shapes, we'll mostly look at sinusoidal waves
- These waveforms can be described by a sine function that changes in both space and time:

$$y(x,t) = A\cos\left(2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right)$$

Longitudinal Wave Represented as a Sine Curve

- A longitudinal wave can also be represented as a sine curve
- Compressions correspond to crests and stretches correspond to troughs
- Also called density waves or pressure waves



Speed of a Wave on a String

- The speed on a wave stretched under some tension, F

$$v = \sqrt{\frac{F}{\mu}} \quad \text{where} \quad \mu = \frac{m}{L}$$

  - μ is called the linear density
  - F is the tension force in the string
- The speed depends only upon the properties of the medium through which the disturbance travels

Example: A violin string playing an "A" note vibrates at a frequency of 440 Hz. It has a linear density of μ = 7 x 10^{-4} kg/m.

- What is the wave speed if the string is under a tension of 10 N?
- What is the wavelength in the string?
- What is the wavelength of the 440 Hz sound wave travelling through the air?

Speed of sound waves

- "Room temperature" air allows sound waves to travel at a speed of about 343 m/s.
- Temperature of air can change the speed of sound!
  - General rule of thumb: sound travels more slowly in cold air, faster in hot air
  - Determine distance to a lightning strike by counting seconds until you hear the thunder (every 5 s ~ 1 mile)
  - Unless otherwise specified, assume room temperature air.

Sound waves:

- Human range of hearing is from about 20-20,000 Hz
- Frequency → pitch