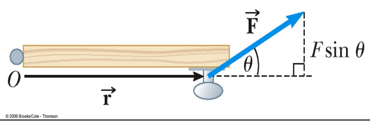
Torque

- Forces are not always perpendicular to the lever arm! Torque definition "picks out" the perpendicular **component** of the force.
- $\tau = r_{perp}F$ or $= rF_{perp}$
- Torque is a vector quantity, can be treated in the same way as forces.



Torque Example

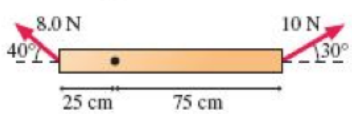
A plumber attempts to loosen a bolt by pushing straight downward on a wrench. If she can exert 75 N of force, compare the torques in two scenarios: one where the wrench is at an angle of 30° above the horizontal, and the other where the wrench is completely horizontal.

Which torque is stronger?

How much stronger is it (by what factor)?

What is the net torque of the bar shown, about the axis indicated by the dot?

$$\tau = rF_{\perp}$$



Revisiting Newton's Laws

1: Need a linear force to change an object's linear motion → Need a torque to change an object's rotational motion

- Equilibrium/Statics:
  - Linear: $\Sigma F = 0$
  - Rotational: $\Sigma \tau = 0$

2: Translational acceleration ~ force, and ~ 1/mass → Angular acceleration ~ torque, and ~ 1/rotational inertia

Newton's Second Law for a Rotating Object

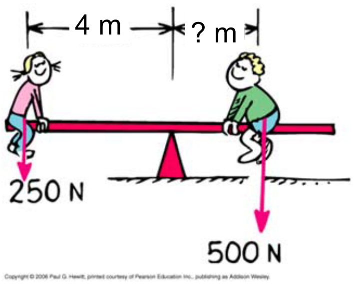
$$\Sigma\tau = I\alpha$$

- The angular acceleration is directly proportional to the net torque
- The angular acceleration is inversely proportional to the moment of inertia of the object
- Combine with **F** = m**a** for full statics problems

Statics problems

- New method for Free Body Diagram: draw WHERE forces act.
- AFTER diagram, decide on coordinate axes
- Write Newton's second law for forces AND torques.

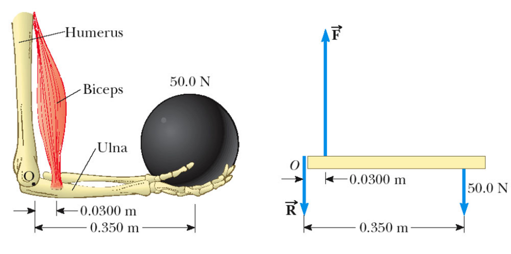
Example: See-Saw Balancing



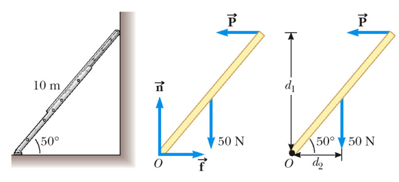
Problem-Solving Strategy for Torque Problems

1. Sketch
2. Free Body Diagram
3. Write out Newton's 2nd law for x, y, and rotation
4. Look at known and unknown information, make algebra plan
5. Solve equations
6. Gut check your answer

Example of a Free Body Diagram (Forearm)



Example of a Free Body Diagram (Ladder)



- The free body diagram shows the normal force and the force of static friction acting on the ladder at the ground
- The last diagram shows the lever arms for the forces

Statics Example 1: Cat on Bench

A cat walks along a uniform plank that is 4.00 m long and has a mass of 7.00 kg. The plank is supported by two sawhorses, one 0.440 m from the left end of the board and the other 1.50 m from the right end. When the cat reaches the right end, the plank just barely begins to tip. What is the mass of the cat?

Statics Example 2: Ladder

An 85.0-kg person stands on a lightweight ladder as shown on the board. The floor is rough and so provides both a normal force and a frictional force. The wall is smooth and exerts only a normal force. Using the dimensions given, find the magnitudes of f_1, f_2, and f_3.

Center of Mass

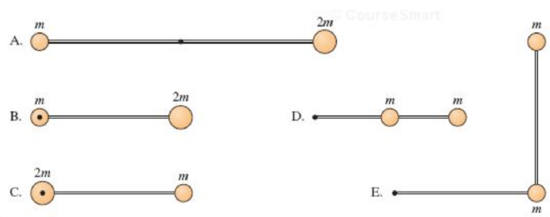
- Average position of all the mass in an object is called the center of mass of object.
- Weight acts at the center of mass → important for torque!

Calculating CM

$x_{cm} = x_1m_1 + x_2m_2 + \ldots/(m_{tot})$
$y_{cm} = y_1m_1 + y_2m_2 + \ldots /(m_{tot})$

Center of mass/center of gravity can also be found experimentally by balancing/hanging an object so that it does not rotate.

Exercise: Where is the CM for each of these objects? Assume rods are massless and 50 cm long, m= 1kg.



Oscillations and Simple Harmonic Motion (SHM)

- Focus on periodic (repeating) motion
- Behavior or motion is described by sine or cosine functions
- Frequency, period characterize systems
- Pendulum, Mass-on-a-spring, uniform circular motion, and waves

Simple Harmonic Motion (SHM)

- An oscillation that can be described by a sinusoidal (sine or cosine) function:

$x(t) = A \cos (2\pi f\, t)$

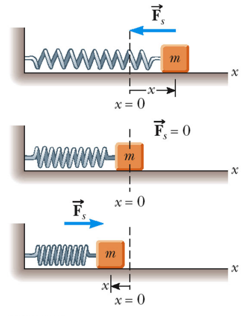
The frequency (f) and amplitude (A) depend on the physical properties of the oscillating system.

Recall Hooke's Law

- Force on a spring: $\mathbf{F} = -k\mathbf{x}$
- Imagine what occurs when we draw a mass on a spring back and let it go (assuming no friction for now).

Hooke's Law Applied to a Spring – Mass System

- When x is positive (to the right), F is negative (to the left)
- When x = 0 (at equilibrium), F is 0
- When x is negative (to the left), F is positive (to the right)

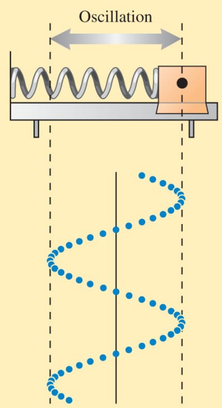


Hooke's Law Force

- The force always acts toward the equilibrium position
  - It is called the *restoring force*
- The direction of the restoring force is such that the object is being either pushed or pulled toward the equilibrium position

Simple Harmonic Motion

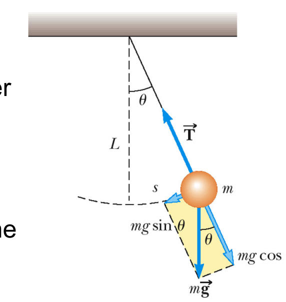
A cart attached to a spring oscillates back and forth. A graph of its motion is sinusoidal. This is a special kind of motion called **simple harmonic motion.**



An understanding of simple harmonic motion will build on many topics from past chapters.

Simple Pendulum

- The simple pendulum is another example of simple harmonic motion
- The force is the component of the weight tangent to the path of motion
  - $F_t = - m g \sin \theta$



Simple Pendulum, cont

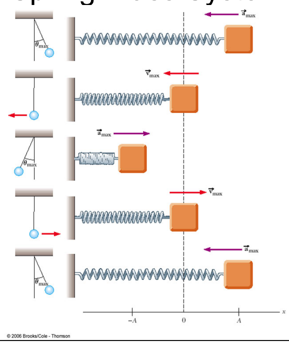
- In general, the motion of a pendulum is not simple harmonic
- However, for small angles, it becomes simple harmonic
  - In general, angles < 15° are small enough
  - $\sin \theta = \theta$
  - $F_t = - m g \theta$
    - This force obeys Hooke's Law

Period of Simple Pendulum

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}} \qquad T = 2\pi\sqrt{\frac{L}{g}}$$

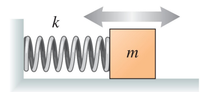
- This shows that the period is independent of the amplitude
- The period depends on the length of the pendulum and the acceleration of gravity at the location of the pendulum

Simple Pendulum Compared to a Spring-Mass System

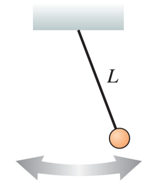


Frequency and Period

The frequency of oscillation depends on physical properties of the oscillator; it does not depend on the amplitude of the oscillation.



$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$



$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$